COMBINED INDUCED TRAPEZOIDAL FUZZY COGNITIVE MAPS

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ABSTRACT

Fuzzy Cognitive Maps is a power full tool which has been used in many fields. The new fuzzy model Combined Induced Trapezoidal Fuzzy Cognitive Maps(COBITpFCMs) was introduced in this paper to analyze the unsupervised data, to combine the expert's opinion, to get the triggering pattern and ranking of the attributes together. And it is illustrated by an empirical example.

KEYWORDS: *FCMs, Trapezoidal Number, Linguistic variable.*

I. Introduction

Fuzzy Cognitive Maps[1] was introduced by Lofti A.Zadeh in the year 1965. Political Scientist R. Axelrod, introduced cognitive maps (CMs) [3] in the year 1975 to represent social scientific knowledge and described the methods that are used for decision making in social and political systems. Then B. Kosko[5, 6] enhanced the power of CMs considering fuzzy values for the concept of the CM and fuzzy degrees of interrelationships between concepts. To represent the unsupervised data fuzzy numbers are playing an important role. In decision making, data analysis and artificial intelligent many models for ranking like Dematel, TOPSIS and AHP are available. Our model deals with the ON-OFF position of the attributes, Triggering pattern, ranking and to combine the experts opinion. When compared to other models, it is observed that the algorithm used in this model is easier. The rest of this paper is organized as follows. Section 2 gives the preliminaries. In Section 3, the definitions and the proposed algorithm of Combined Induced Trapezoidal Fuzzy Cognitive Maps (COBITpFCMs) is given. Section 4 provides numerical example. Finally, Section 5 concludes the paper.

II. Preliminaries

2.1. Trapezoidal fuzzy number and the algebraic operation

2.2.1. Trapezoidal fuzzy number

A Trapezoidal fuzzy number A with four parameters $a_1 < a_2 < a_3 < a_4$ is denoted as

$A = (a_1, a_2, a_3, a_4)$ in the set of real numbers R (see Fig. 1).

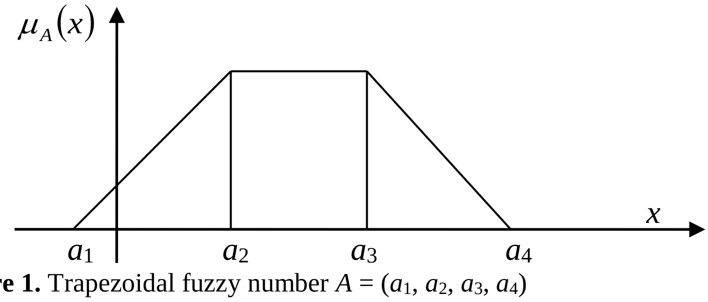

Figure 1. Trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$

The membership function of Trapezoidal fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & , x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & , a_1 \le x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4} & , a_2 \le x \le a_3 \\ 0 & , a_4 < x \end{cases}$$

2.2.2. Operations of Trapezoidal fuzzy number

Let $A_1 = (a_{11}, a_{12}, a_{13}, a_{14})$ and $A_2 = (a_{21}, a_{22}, a_{23}, a_{24})$ be two Trapezoidal fuzzy number in the set of real numbers R. Then the following are the operations that can be performed on Trapezoidal fuzzy number.

(i) Addition:

$A_1 + A_2 = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24})$

(ii) Subtraction:

$A_1 - A_2 = (a_{11} - a_{24}, a_{12} - a_{23}, a_{13} - a_{22}, a_{14} - a_{21})$.

2.2.3. Degrees of the Trapezoidal fuzzy number

The linguistic values of the Trapezoidal fuzzy numbers are given in Table 1 [4].

Table 1. The linguistic values of the Trapezoidal fuzzy numbers.

Linguistic Term	Linguistic Values of the Trapezoidal Fuzzy Number
Very Low	(0.0, 0.0, 0.0, 0.0)
Low	(0.0, 0.1, 0.2, 0.3)
Fairly Low	(0.2, 0.3, 0.4, 0.5)
Fair	(0.4, 0.5, 0.6, 0.7)
Fairly high	(0.6, 0.7, 0.8, 0.9)
High	(0.8, 0.9, 1,1)
Very High	(1, 1, 1, 1)

III. Trapezoidal Fuzzy Cognitive Maps

TpFCMs are more applicable when the data in the first place is an unsupervised one. The TpFCM models the world as a collection of classes and causal relations between classes.

3.1. Basic Definitions of TpFCM

Definition 3.1.1. When the nodes of the TpFCMs are fuzzy sets then they are called as Fuzzy Trapezoidal nodes.

Definition 3.1.2. TpFCMs with edge weights or causalities from the set {−1, 0, 1} are called simple TpFCMs.

Definition 3.1.3. A TpFCM is a directed graph with concepts like policies, events, etc., as nodes and causalities as edges. It represents causal relationship between concepts.

Definition 3.1.4. Consider the nodes/concepts $TpC_1, TpC_2, TpC_3, \ldots, TpC_n$ of the TpFCMs. Suppose the directed graph is drawn using edge weight $Tpe_{ij} \in \{-1, 0, 1\}$. The Trapezoidal matrix M be defined by Tp(M) = (Tpe_{ij}) where Tpe_{ij} is the Trapezoidal weight of the directed edge $\overrightarrow{TpC_i TpC_j}$.

Tp(M) is called the adjacency matrix of TpFCMs, also known as the connection matrix of the TpFCM. It is important to note that all matrices with a TpFCM are always square matrices with zero as diagonal entries.

Definition 3.1.5. Let TpC_1, TpC_2, TpC_3, …, TpC_n be the nodes of a TpFCM. $A = (a_1, a_2, a_3, a_4)$ where Tpe_{ij} ε {−1, 0, 1}. *A* is called the instantaneous state vector and it denotes the ON–OFF position of the node at any instant.

$$\text{Instantaneous vector} = \begin{cases} Tpa_i = 1, Maximum\ \ weight \\ Tpa_i = 0, Otherwise \end{cases}$$

Definition 3.1.6. Let TpC_1, TpC_2, TpC_3, …, TpC_n be the nodes of a TpFCM. Let $\overrightarrow{TpC_1TpC_2}$, $\overrightarrow{TpC_1TpC_2}$, …$\overrightarrow{TpC_iTpC_j}$, be the edges of the TpFCM (i≠j). Then the edges form a directed cycle. A TpFCM is said to be cyclic if it possesses a directed cycle. A TpFCM is said to be acyclic if it does not possess any directed cycle.

Definition 3.1.7. A TpFCM with cycles is said to have a feedback.

Definition 3.1.8. When there is a feedback in a TpFCM, i.e., when the causal relations flow through a cycle in a revolutionary way, the TpFCM is called a dynamical system.

Definition 3.1.9. Let $\overrightarrow{TpC_1TpC_2}$, $\overrightarrow{TpC_1TpC_2}$, …, $\overrightarrow{TpC_iTpC_j}$, be a cycle. When TpC_i is switched on and if the causality flows through the edges of a cycle and if it again causes TpC_i, we say that the dynamical system goes round and round. This is true for any node, for i = 1,2,3, … , n. the equilibrium state for this dynamical system is called the hidden pattern.

Definition 3.1.10. If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. Consider a TpFCM with TpC_1, TpC_2, TpC_3, …, TpC_n as nodes. For example, let us start the dynamical system by switching on TpC_1 and assume that the TpFCM settles down with TpC_1 and TpC_n i.e., the state vector remains as (1, 0, 0, …, 0, 1). This state vector is called the fixed point.

Definition 3.1.11. If the TpFCM settles down with a state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow \dots A_i \rightarrow A_1$, then this equilibrium is called limit cycle.

Definition3.1.12. Weighted average of a fuzzy number (a_1,a_2,a_3,a_4), with the weight (z_1,z_2,z_3,z_4) is $((a_1).(z_1)+(a_2).(z_2)+(a_3).(z_3)+(a_4).(z_4))/(z_1+z_2+z_3+z_4)$

3.2. The proposed Algorithm

To get the triggering pattern of each attribute and ranking of the attributes, these steps are to be followed:

Step – 1: Let TpC_1, TpC_2, TpC_3, …, TpC_n be the nodes of a TpFCM with feedback.

Step – 2: Get the experts Opinion (adjacency matrix) Tp(M) from 5 experts.

Step – 3: Let the weight of the experts be (z_1,z_2,z_3,z_4)

Step – 3: Find the Average

(1) Apply the Numerical values of the Linguistic values.
(2) Combine the five experts opinion using the weighted arithmetic average formula(3.1.12) , which is the adjacency matrix Tp(M) .

Step – 4: Find the hidden pattern when TpC_1 is switched ON, consider A= (1 0 0 0 0 0 0 0 0 0).

(i) Find A*Tp(M) and denote it as $ATp(M)_{weight}$.
(ii) Find average of each linguistic value and denote it as $ATp(M)_{weight(Avg)}$.

(iii) Threshold the vector by replacing a_i by 1 if a_i is the maximum weight of the trapezoidal node, otherwise replace a_i by 0.
(iv) Form different vectors for all 1's in which the previous value at the position of 1 should be reset for that 1, and find the product of each with the given matrix.
(v) The vector which has maximum number of maximum weight is considered as A_2.
(vi) Repeat the same process till $A_i = A_{i+1}$ which is the fixed point.

Step – 5: Find the Hidden pattern for all Trapezoidal nodes.
Step – 6: Find the Total Average weight of each attribute.

IV. Example

The survey was conducted in various old age homes in Chennai. We framed a linguistic questionnaire and administered the same to 100 old aged persons living under different difficult circumstances. Listed are the 10 problems based on our interviews and surveys to analyze the major problem of the old age people:

TpC_1 – Neglected

TpC_2 – Treated as Burden

TpC_3 – Forced to sell their property

TpC_4 –Lack of care

TpC_5 – Abandoned

TpC_6 – Lack of reason to live

TpC_7 – Depression and trauma

TpC_8 – Isolation

TpC_9 – Lack of emotional support

TpC_{10} – Lack of economical support

4.1Application of COBITpFCM

These 10 attributes are divided into 6 classes CS1, CS2, ..., CS5 with 5 in each class. Let CS1 = {C1, C2, C3, C14, C15}, CS2 = {C3, C4, C6, C8, C9}, CS3 = {C1, C2, C5, C7, C10}, CS4 = {C3, C4, C5, C7, C10}, CS5 = {C6, C7, C8, C9, C10} and CS6 = {C1, C2, C6, C8, C9}.

The adjacency matrix $Tp(M)_i$ was constructed based on the experts opinion as shown below

Adjacency matrix of first expert

$$Tp(M)_1 = \begin{array}{c|ccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \hline C_1 & VL & F & VL & H & VH \\ C_2 & H & VL & L & H & VH \\ C_3 & L & H & VL & FH & H \\ C_4 & H & VH & FH & VL & H \\ C_5 & L & F & FL & H & VL \end{array}$$

Adjacency matrix of Second expert

$$Tp(M)_2 = \begin{array}{c|ccccc} & C_3 & C_4 & C_6 & C_8 & C_9 \\ \hline C_3 & VL & F & VH & FH & F \\ C_4 & VH & VL & H & FH & VH \\ C_6 & L & L & VL & F & H \\ C_8 & VL & FH & VH & VL & H \\ C_9 & VH & VH & VH & H & VL \end{array}$$

Adjacency matrix of third expert

$$Tp(M)_3 = \begin{array}{c|ccccc} & C_1 & C_2 & C_5 & C_7 & C_{10} \\ C_1 & VL & L & VH & VH & H \\ C_2 & H & VL & VH & H & VL \\ C_5 & FL & L & VL & VH & VH \\ C_7 & F & L & FH & VL & L \\ C_{10} & VL & H & VH & VH & VL \end{array}$$

Adjacency matrix of forth expert

$$Tp(M)_4 = \begin{array}{c|ccccc} & C_3 & C_4 & C_5 & C_7 & C_{10} \\ C_3 & VL & VH & F & L & VL \\ C_4 & VH & VL & H & F & FL \\ C_5 & VH & VH & VL & FL & FH \\ C_7 & VH & H & VH & VL & FL \\ C_{10} & FH & H & H & F & VL \end{array}$$

Adjacency matrix of fifth expert

$$Tp(M)_5 = \begin{array}{c|ccccc} & C_6 & C_7 & C_8 & C_9 & C_{10} \\ C_6 & VL & VH & F & L & VL \\ C_7 & VH & VL & H & F & FL \\ C_8 & VH & VH & VL & FL & FH \\ C_9 & VH & H & VH & VL & FL \\ C_{10} & FH & H & H & F & VL \end{array}$$

Adjacency matrix of sixth expert

$$Tp(M)_6 = \begin{array}{c|ccccc} & C_1 & C_2 & C_6 & C_8 & C_9 \\ C_1 & VL & FL & H & H & VH \\ C_2 & VH & VL & F & FH & H \\ C_6 & VL & FH & VL & H & FH \\ C_8 & FH & H & VH & VL & VH \\ C_9 & FL & FH & VH & VH & VL \end{array}$$

Table 2. Weights of the attributes.

	TpC_1	TpC_2	TpC_3	TpC_4	TpC_5	TpC_6	TpC_7	TpC_8	TpC_9	TpC_{10}
TpC_1	0.00	0.01	0.01	0.01	0.03	0	0.02	0	0	0
TpC_2	0	0	0	0	0	0.01	0	0	0	0
TpC_3	0.01	0	0.01	0.01	0	0	0.02	0	0	0.02
TpC_4	0	0	0	0	0	0	0	0	0	0.01
TpC_5	0	0	0	0	0	0	0.01	0	0	0.01
TpC_6	0	0	0.01	0.01	0	0.01	0	0.03	0.01	0
TpC_7	0	0	0	0	0	0	0.01	0	0	0.01
TpC_8	0.04	0.05	0	0.02	0	0.09	0.01	0	0.08	0.01
TpC_9	0.04	0.05	0	0.02	0	0.09	0.01	0	0.08	0.01
TpC_{10}	0	0	0	0	0.01	0	0	0	0	0
Total	0.09	0.11	0.03	0.07	0.04	0.20	0.08	0.03	0.17	0.07
Average	0.009	0.011	0.003	0.007	0.004	0.020	0.008	0.003	0.017	0.007
Rank	4	3	9	6	8	1	5	9	2	6

By this model it was observed that Lack of reason to live and Lack of emotional support are the major problems of old age people as shown in Table 2.

TABLE 3: Triggering pattern

S. No.	Attribute ON state	Triggering Pattern
1	(1 0 0 0 0 0 0 0 0 0)	$TpC_1 \rightarrow TpC_5 \rightarrow TpC_5$
2	(0 1 0 0 0 0 0 0 0 0)	$TpC_2 \rightarrow TpC_7 \rightarrow TpC_6 \rightarrow TpC_6$
3	(0 0 1 0 0 0 0 0 0 0)	$TpC_3 \rightarrow TpC_7 \rightarrow TpC_7$
4	(0 0 0 1 0 0 0 0 0 0)	$TpC_4 \rightarrow TpC_3 \rightarrow TpC_{10} \rightarrow TpC_{10}$
5	(0 0 0 0 1 0 0 0 0 0)	$TpC_5 \rightarrow TpC_7 \rightarrow TpC_7$
6	(0 0 0 0 0 1 0 0 0 0)	$TpC_6 \rightarrow TpC_8 \rightarrow TpC_8$
7	(0 0 0 0 0 0 1 0 0 0)	$TpC_7 \rightarrow TpC_5 \rightarrow TpC_7$
8	(0 0 0 0 0 0 0 1 0 0)	$TpC_8 \rightarrow TpC_6 \rightarrow TpC_8$
9	(0 0 0 0 0 0 0 0 1 0)	$TpC_9 \rightarrow TpC_6 \rightarrow TpC_8$
10	(0 0 0 0 0 0 0 0 0 1)	$TpC_{10} \rightarrow TpC_7 \rightarrow TpC_5 \rightarrow TpC_7$

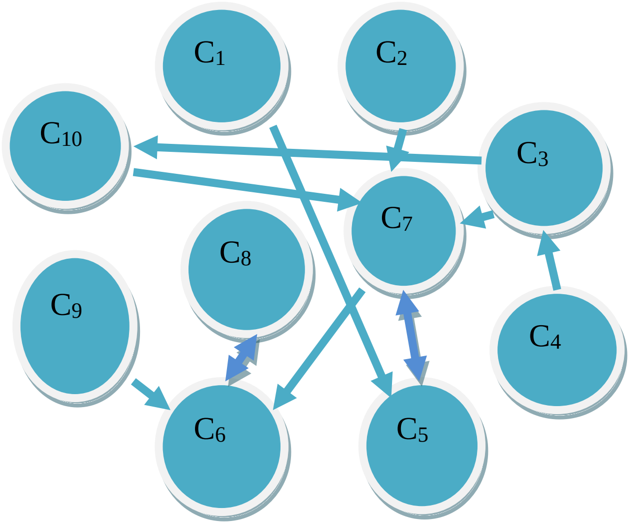

Figure 2. Triggering Pattern graph.

V. Results and Discussion

It is observed that TpC_7 and TpC_8 are the terminal nodes. Also it is shown that all attributes trigger TpC_7 and TpC_8 (see Fig. 2).

By this new model COBITpFCMs, the problems of old age people are analyzed. $TpC_6 > TpC_9 > TpC_2 > TpC_1 > TpC_7 > TpC_4$, $TpC_{10} > TpC_5 > TpC_3 > TpC_8$ is the ranking of the attributes. It shows that the major causes of the old age people are TpC_6 - Lack of reason to live and TpC_9 – Lack of emotional support. The attributes TpC_8 – Isolation and TpC_3 – Forced to sell their property are not affecting old age people to a large extent. The Triggering pattern shows that all attributes induce the attributes TpC_7 and TpC_8.

VI. Conclusion

To analyze the unsupervised data, to combine the expert's opinion, to get the triggering pattern and ranking of the attributes together, A new fuzzy model, Combined Induced Trapezoidal Fuzzy Cognitive Maps(COBITpFCMs) was introduced in this paper And it is illustrated by an empirical example.

10 problems of old age people were taken, based on our interviews and surveys to analyze the major problem of the old age people. These 10 attributes are divided into 6 classes CS1, CS2, ..., CS5 with 5 in each class. Let CS1 = {C1, C2, C3, C14, C15}, CS2 = {C3, C4, C6, C8, C9}, CS3 = {C1, C2, C5, C7, C10}, CS4 = {C3, C4, C5, C7, C10}, CS5 = {C6, C7, C8, C9, C10} and CS6 = {C1, C2, C6, C8, C9}. The adjacency matrix Tp(M)i was constructed based on the experts opinion.

From this example, it was observed that TpC7 and TpC8 are the terminal nodes. Also it was shown that all attributes trigger TpC7 and TpC8 . Also TpC6> TpC9> TpC2 >TpC1 > TpC7> TpC4, TpC10 >TpC5 > TpC3 > TpC8 is the ranking of the attributes. It shows that the major causes of the old age people are TpC6 - Lack of reason to live and TpC9 – Lack of emotional support. The attributes TpC8 – Isolation and TpC3 – Forced to sell their property are not affecting old age people to a large extent. The Triggering pattern shows that all attributes induce the attributes TpC7 and TpC8.

From this example it is clear that this model gives the hidden pattern, ranking and triggering pattern of the attributes, by simple calculation. The advantage of this method is less time consuming.

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