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THE ANALYSIS OF VIBRATIONAL RESPONSE OF STRUCTURES WITH UNCERTAIN PARAMETERS

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ABSTRACT

This paper is based on study of vibrational responses of different vibrating systems at different frequency with uncertain parameters. The concept of uncertainty plays an important role in the design of practical mechanical system. So it becomes important to study its effects on mechanical system for different frequency domain i.e. low, medium and high frequency. There are large and varied amount of research has been dedicated to developing techniques which predict the dynamic responses of structures in all frequency domain and structures with uncertainty. Here in this paper structural element plate is selected on which mass and stiffness uncertainty are taking in account. By using FEM technique finding out, how plate is dynamically behaves in vibration.

KEYWORDS: *Uncertain Parameters, Vibration analysis, Modal Analysis, FEM and Ansys software.*

I. INTRODUCTION

Predicting the dynamic response of a vibrating system generally involves determining the equations of motion of the structure and solving them in order to find the natural frequencies and mode shapes of the system for given boundary conditions. The natural frequencies and mode shapes can then be used to predict the response due to an applied excitation. The dynamic equation for a multi-degree of freedom system is given below [1]

$$KX + C\dot{X} + M\ddot{X} = F \qquad (1.1)$$

where K, C and M are respectively the stiffness, damping and mass matrices. X is a vector of the nodal degrees of freedom and F is the load vector

For more complex systems, the equations of motion can be approximated using various deterministic modelling techniques such as finite element analysis (FEA) and dynamic stiffness techniques. These methods are extensively used to predict the linear dynamic response of structures in the low frequency region. Finite element methods are one of the most widely used deterministic techniques. For this technique, a structure is divided into a number of elements. The dynamic behaviour of each element is derived and the solution of the structure as a whole is reconstituted by attaching each element together at its node points. Energy methods such as Statistical Energy Analysis (SEA) are appropriate dynamic predictive techniques at high frequencies. SEA is a technique which models a structure in terms of its gross system properties. In this method, a complete system is described in terms of energy flow between various subsystems [2].

In engineering design, it is important to calculate the response quantities such as the displacement, stress, vibration frequencies, and mode shapes of given set of design parameters. The study of mathematical models which involve physical and geometric parameters such as mass density ρ, elastic modulus E, Poisson's ratio v, lengths, and cross-section shape characteristics. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution geometric properties or physical errors, as well as variation arising from the assembly and manufacturing processes. In engineering design these uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered. This concept of uncertainty plays an important role in investigation of various engineering problems.

1.1Research Overview

This paper is concerned with observing and predicting the dynamic characteristics and response of structures with uncertainties. In literature review section different papers related to dynamic characteristics and responses of structures with uncertainty are studied. In Mathematical modeling section equation of motion for a bare plate and a plate with mass and stiffness uncertainty is derived. In analysis of bare rectangular plate section analysis is done by using Ansys software and finding different value of natural frequency. In predicting frequency response function section, finding the response of bare rectangular plate by plotting FRF on frequency vs amplitude graph.

II. LITERATURE REVIEW

In the case studies of Sondipon Adhikari, Michael I. Friswell and Kuldeep P. Lonkar uncertainty in structural dynamics experimental on beams and plates paper has described two experiments that may be used to study methods to quantify uncertainty in the dynamics of structures.1) The fixed-fixed beam is very easy to model and the results of a one hundred sample experiment with randomly placed masses are described in this paper. 2) The cantilever plate experiment with randomly located oscillators with randomly varying stiffness is designed to simulate model uncertainty [1]. A. Cicirello, R. S. Langley done the analysis of random systems with combined parametric and non-parametric uncertainty models is an extension to the hybrid FE-SEA method. In which parametric (probabilistic pdf and interval analysis) and non-parametric modelling of uncertainties are combined together [2]. Using the convex model theory, a new method to optimize the dynamic response of mechanical system with uncertain parameters is derived and there is no need for probability density functions [3]. G. Manson calculate frequency response functions for uncertain systems using complex affine analysis. The methods are demonstrated on the problem of calculation of the frequency response functions of a simple lumped mass system [4].
The extreme value distribution and dynamic reliability analysis of nonlinear structures with uncertain parameters. A new approach for evaluation of the extreme value distribution and dynamic reliability assessment of nonlinear structures with uncertain parameters is proposed [5].Interval optimization for uncertain structures, this paper presents an interval optimization method to solve the uncertain problems of the vibration systems with multi-degrees of freedom, where the structural characteristics are assumed to be expressed as interval parameters [6].Uncertainties and dynamic problems of bolted joints and other fasteners. This review article provides an overview of the problems pertaining to structural dynamics with bolted joints [7]. Modal analysis of structures with uncertain-but-bounded parameters via interval analysis by using modal analysis and interval calculus, investigated the method of computing upper and lower bounds of parameters such as, natural frequencies, modal shapes [8]. A. Pratellesia, M. Viktorovitchb, N. Baldanzinia, and M. Pierini done a new formulation able to predict the behaviour of structures in the mid-frequency range is presented in this paper. The mid-frequency field is a hybrid domain for which assembled structures exhibit simultaneously low- and high-frequency behaviours, depending on the material and geometrical properties of different subsystems [9].
A hybrid method combining FE and SEA was recently presented by Robin Langley for predicting the steady-state response of vibro-acoustic systems with uncertain properties. The subsystems with long wavelength behavior are modelled deterministically with FE, while the subsystems with short wavelength behavior are modelled statistically with SEA, [10]. In 2011 Giuseppe Quaranta presents the finite element analysis method by taking into account probability density functions whose parameters are affected by fuzziness. It is shown that the proposed methodology is a general and versatile tool for finite element analyses because it is able to consider, both, probabilistic and non-probabilistic sources of uncertainties, such as randomness, vagueness, ambiguity and imprecision [11]. In 2012 A.L. Morales, J.A.Rongong and N.D.Sims used a fuzzy design method in the finite element procedure to simulate and analyze active vibration control of structures subjected to uncertain parameters [12]. Based on the interval random model, the hybrid uncertain acoustic dynamic equilibrium equation is constructed and a hybrid uncertain analysis method is proposed. In the

proposed method, the interval random matrix and vector are expanded by the first-order Taylor series and the sound pressure response vector is obtained by the matrix perturbation technique [13].

III. MATHEMATICAL MODELING

All the Vehicles, aircraft and home appliances structures are made up of Plate or combination of Plates so it becomes necessary to study Plate vibration. Using the Lagrange- Rayleigh -Ritz technique [15], the equations of motion of a dynamic system in modal space can be derived.

3.1. Bare Rectangular Plate

Considering the simply supported bare rectangular plate (with no structural uncertainty). For this plate eigenfunction is described by sinusoidal mode shapes in the *x* and *y* directions, respectively [15]

$$\phi_{mn}(x) = \phi_m(x)\phi_n(y) \qquad (2.1)$$

$$\phi_m(x) = \sin(m\pi x / L_x) and \phi_n(y) = \sin(n\pi x / L_y) \qquad (2.2)$$

The flexural displacement of a bare rectangular plate in modal space is given by [15]

$$w(x, y, t) = \sum_{mn} q_{mn}(t)\psi_{mn}(x) \qquad (2.3)$$

Where, q is the modal coordinate and m, n are the mode numbers of the shape functions in the x and y directions, respectively and;

$$\psi_{mn}(x) = \psi_m(x)\psi_n(y) \qquad (2.4)$$

are the mass-normalized eigenfunctions which satisfy the following orthogonality condition[16]

$$\int_0^{L_x}\int_0^{L_y} \rho h\, \psi_{mn}\psi_{m'n'}\, dxdy = \{^{1 \;\; mn=m'n'}_{0 \;\; mn\neq m'n'} \qquad (2.5)$$

L_x and L_y are respectively the lengths of the plate in the x and y directions, h are the plate thickness and ρ is the density

For a plate simply supported on all four sides, the mass normalized eigenfunctions are given by

$$\psi_{mn} = \frac{1}{M_n}\phi_{mn}(x) = \frac{1}{M_n}\sin\left(\frac{m\pi x}{L_x}\right)\sin\left(\frac{n\pi y}{L_y}\right) \qquad (2.6)$$

Where $M_n = \rho h L_x L_y / 4$ is the modal mass.

Using the orthogonality condition, an expression for the kinetic energy of a bare plate becomes

$$\begin{aligned} T &= \frac{\rho h}{2}\int_0^{L_x}\int_0^{L_y} \dot{w}^2(x)\, dxdy \\ &= \frac{\rho h}{2}\sum_{mn}\sum_{jk} \dot{q}_{mn}\dot{q}_{jk}\psi_{mn}(x)\psi_{jk}(x) \\ &= \frac{1}{2}\sum \dot{q}_{mn}{}^2 \end{aligned} \qquad (2.7)$$

Where $\dot{w}$denotes the derivative of w with respect to time

Expression for the potential energy of the plate can be obtained as

$$V = \frac{1}{2}\sum_{mn} \omega_{mn}^2 q_{mn}^2 \qquad (2.8)$$

Where; $\omega_{mn} = \sqrt{\frac{D}{\rho h}}\left(\left(\frac{m\pi}{L_x}\right)^2 + \left(\frac{n\pi}{L_y}\right)^2\right) \qquad (2.9)$

corresponds to the natural frequencies of the bare plate and

$$D = \frac{Eh^3}{12(1 - v^2)} \qquad (2.10)$$

is the plate flexural rigidity and E, v are respectively Young's modulus and Poisson's ratio

Lagrange's equation for a particular modal coordinate j is given by [15]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}+\frac{\partial V}{\partial q_j}=0, \quad j=1,2......N \tag{2.11}$$

Differentiating the kinetic and potential energies with respect to the modal coordinate and substituting into Lagrange's equation results in the equation of motion of the bare plate.

$$\ddot{q}+\omega^2 q=0 \tag{2.12}$$

The natural frequencies can then be obtained by eigenvalue analysis [20].

3.2. Uncertain Mass-and-Spring-Loaded Plate

Now consider a mass-and-spring-loaded uncertainty on the plate as shown in Figure 1.

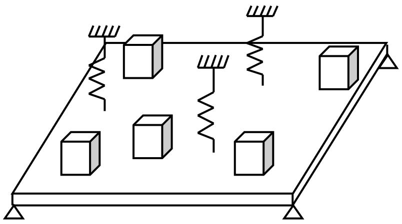

Figure1. Plate Showing Loaded Mass and Spring Uncertainty

For these uncertainties the equation of motion is changes as follows [17].

$$\begin{aligned}&\ddot{q}_{pq}+\sum_{N_m}\sum_{mn} m_a \ddot{q}_{mn}\psi_{mn}(x_m)\psi_{pq}(x_m)\\&+\sum_{N_m}\sum_{mn} kq_{mn}\psi_{mn}(x_k)\psi_{pq}(x_k)+\omega_{pq}^2 q_{pq}=0\end{aligned} \tag{3.1}$$

Where; N_m number of point masses and N_k springs to ground (of stiffness k), are the mass-normalized eigenfunctions , and x_m , x_k respectively correspond to the random locations of the added masses and springs.

IV. Analysis of Bare Rectangular Plate in Ansys

A rectangular plate of dimension 500mm×600mm×2mm of steel material with properties of ρ=7.86×10^{-9},ν=0.3 ,Y=2e^5 MPa was created .Applying all material properties and boundary conditions to plate whose all edges are simply supported. Selecting the element type quadratic shell element 181. SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. The degenerate triangular option should only be used as filler elements in mesh generation. Shell 181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses [21]. Figure 2 showing discretize plate model with meshing and number of nodes and element. The range is from 0 Hz to 300Hz. The solver used is Block Lanczos [21] to find its natural frequency values as shown in the Table 1 and exited its mode shape. In figure 3(a) value of natural frequency at node 1 is 32.48 Hz and similarly in figure 3(b) value of natural frequency at node 3 is 90.16 Hz with their mode shapes.

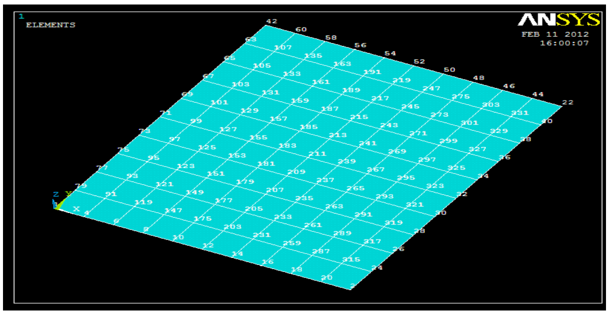

Figure 2. Discretize plate model

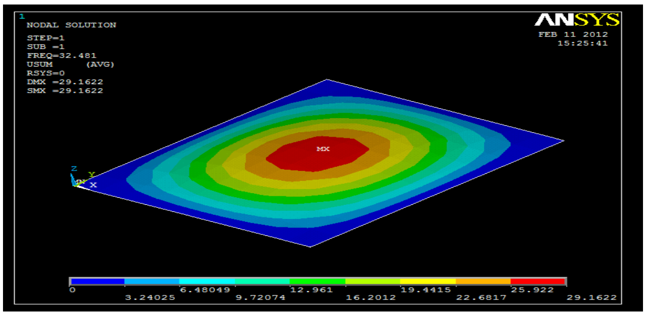

Figure 3(a). Natural Frequency at Node 1

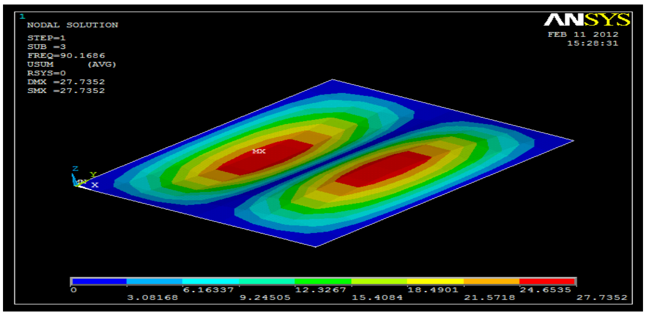

Figure 3(b). Natural Frequency at Node 3

Table 1. Different value of natural frequency

Node No.	Natural Frequency(Hz)
01	32.48
02	72.53
03	90.16
04	130.4
05	139.47
06	186.54
07	198.20
08	227.38
09	234.23
10	296.00

V. PREDICTING FREQUENCY RESPONSE FUNCTION

To know FRF plot harmonic analysis is to be done [18]. For the same plate apply the force of 1N at node 211 and giving the range of natural frquency of 0 Hz to 300Hz as shown in Figure 4. It will generate FRF plot on graph of amplitude (mm) to frequency(Hz) as shown in Figure 5. It is time domain plot . Here getting signal in verticle direction in time domain .Same range is transmitted in frquency domain to incerse its redabilyty and clearly see the picks at different frquency.

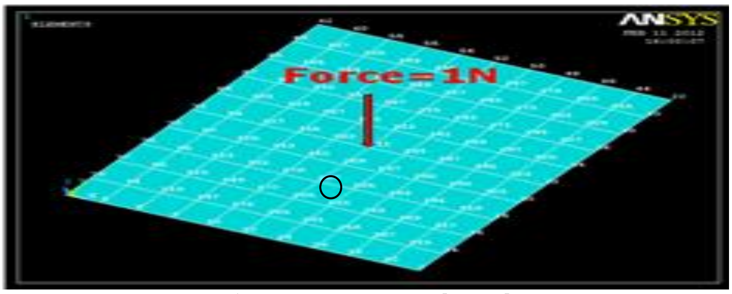

Figure 4. Force Loading Plate

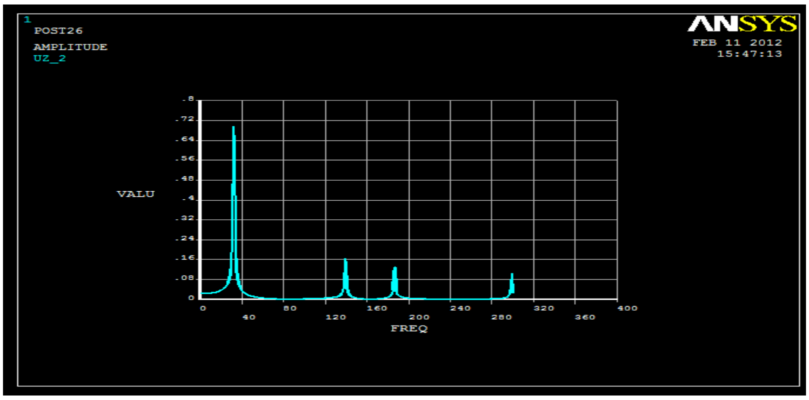

Figure 5. FRF Plot Amplitude (mm) vs Frequency(Hz**)**

To know the response in more details need to plot it on log scale as shown in Figure 6. It is harmonic response of plate at node no 2 in log scale. Here it is clearly showing node and antinode points which is missing in time domain signal [19]. Similarly in Figure 7 harmonic response of plate at node no 3 on log scale is shown. So it is ploted in frquency domain to get clear veiw of it on log scale to get all pickes clearly visible and repeat for different location. This is overall response of plate at different location . This gives quite important information for plate natural frquencies , mode shapes and harmonic response on time scale as well as on log scale.

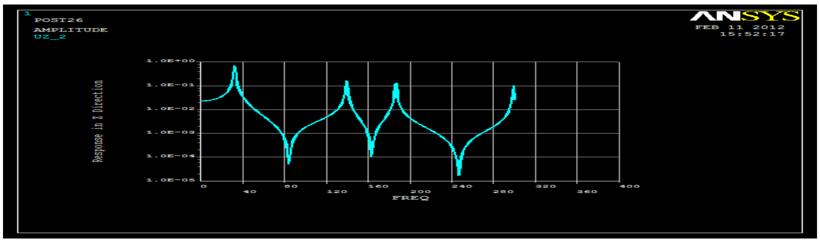

Figure 6. FRF on Log Scale at Node 2

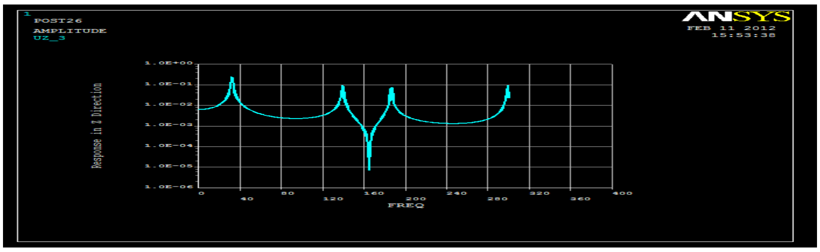

Figure 7. FRF on Log Scale at Node 3

VI. CONCLUSION

As on the linear scale of amplitude (mm) vs. frequency (Hz) graph as shown in Figure 5, there are four picks found. First pick found at 32.48 Hz natural frequency and amplitude is 0.71mm respectively. So here the value of amplitude is highest i.e. nothing but resonance point. With this it comes to know that displacement response with respect to frequency.
Now, to know more details of this it should be plotted on log scale as shown in Figure 6. From this graph come to know its resonance point and anti-resonance point. Also many details of system such as amount of displacement, by how much frequency by how much amount system excited, damping constant and it can say overall response of system can be known.

All this conclusions are for the bare rectangular plate, so for a plate with mass and spring uncertainty add it to the plate as shown in Figure1 and in the same way analyzed it and compare with bare plate response and see how response will be deviate.

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