

A STUDY OF CHANNEL ESTIMATION TECHNIQUES WITH CARRIER-FREQUENCY OFFSET ESTIMATION IN SISO-OFDM SYSTEMS

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ABSTRACT

The channel estimation techniques for OFDM systems based on pilot arrangement are investigated. That on this basis pilots were inserted among subcarriers in transmitter with distances emerged of sampling theory. The objective of this study is improving channel estimation accuracy in OFDM systems because channel state information is required for signal detection at receiver and its accuracy affects the overall performance of system and it is essential to improve the channel estimation for more reliable communications. For improving the quality of channel estimation, different iterative algorithms are available, such as EM, RLS and LMS, among them LMS is chosen due to its less complexity comparing with other methods and its acceptable performance. The low complexity proposed receiver including LMS algorithm, has a higher efficiency than conventional methods and it can work in lower amount of SNRs. According to this, using maximum likelihood estimation algorithm (MLE), the carrier frequency offset (CFO) can be estimated. Then we estimate channel's coefficients for these systems. To accurately estimate the channel's coefficients, carrier frequency offset mitigation is necessary. The efficiency of these algorithms can be investigated with simulation and the results of estimation will come to a comparison.

KEYWORDS: *Single-Input Single-Output systems, Channel Estimation, Synchronization, LS algorithm, LMS algorithm, Carrier frequency offset*

I. INTRODUCTION

In orthogonal frequency-division multiplexing (OFDM), multiple user symbols are transmitted simultaneously over orthogonal subcarriers which form an OFDM symbol [1],[2]. Compared to other modulation methods, OFDM symbols have a relatively long time duration, whereas each subcarrier has a narrow bandwidth. The bandwidth of each subcarrier is small enough to assume a flat (nonselective) fading in a moderately frequency-selective channel. These subcarriers have overlap in time and frequency domains, nonetheless, the signal waveforms are designed to be orthogonal, due to a cyclic extension of each OFDM symbol in the time domain. A practical implementation involves the inverse fast Fourier transform (IFFT) at the transmitter and the fast Fourier transform (FFT) at the receiver. The narrowband nature of subcarriers makes the signal robust against frequency selectivity caused by a multipath delay spread. However, OFDM is relatively sensitive to its dual, namely, time selectivity, which is due to rapid time variations of a mobile channel. Time variations corrupt the orthogonality of the OFDM subcarrier waveforms so that inter carrier interference (ICI) occurs.

Orthogonal frequency division multiplexing (OFDM) techniques has widely been considered to be a very promising strategy to enhance data rate, capacity, and quality for broadband wireless systems over frequency-selective fading channels [3]. Along with this strategy, oscillator jitter and Doppler shift make carrier frequency offset (CFO) that degrades the performance of system remarkably [4].

More recently, due to mutually relation of channel impulse response (CIR) and CFO, joint channel and frequency offset estimation issue have received a lot of attentions in OFDM context [5].

The main goals of synchronization include identifying the beginning of individual OFDM symbols, and ensuring inter carrier orthogonality. Various approaches have been proposed to estimate timing and frequency offsets: either jointly or separately; and either blindly or with the aid of pilot symbols and training sequences.

The frequency offset in the OFDM symbols represents the frequency shift between the transmitted and the received symbols. This case arises as a result of frequency change in the receivers during transmission. As a result of this frequency offset problem, there arises a condition where the orthogonality of the individual carriers is affected. Thus the inter symbol interference (ISI) arises.

There are two deleterious effects caused by frequency offset; one is the reduction of signal amplitude in the output of the filters matched to each of the carriers and the second is introduction of ISI from the other carriers which are now no longer orthogonal to the filter. Because, in OFDM, the carriers are inherently closely spaced in frequency compared to the channel bandwidth, the tolerable frequency offset becomes a very small fraction of the channel bandwidth.

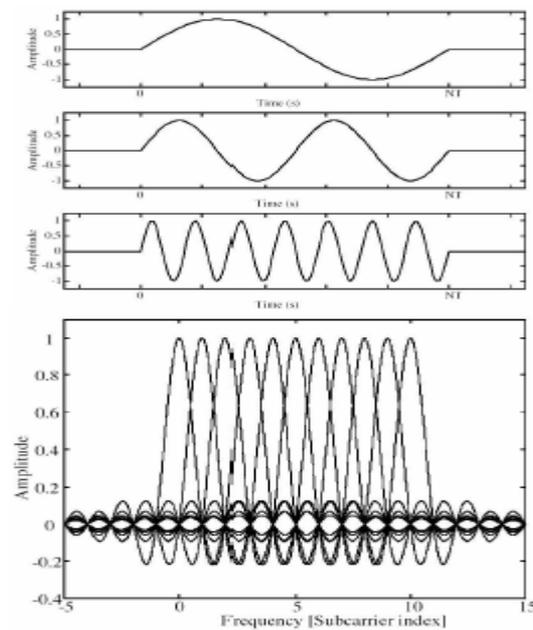


Figure 1. Subcarriers and frequency spectrum of an OFDM signal.

Further, in the second chapter synchronizing methods are introduced. In the third chapter, the maximum likelihood estimation (MLE) algorithm will be presented in order to estimate CFO, in the fourth chapter the Channel's coefficients estimation algorithms in SISO-OFDM systems will be described and finally, the simulation results will indicate the performance of suggested algorithms.

II. SYNCHRONIZATION METHODS

According to the surveys which have done until now, the first article with title synchronization in MIMO-OFDM systems has been published by Mody and Stuber in 2001.[6,7]. In those articles, Mody and Stuber generalized synchronization algorithm, proposed by Schmidl, Cox[8], for OFDM systems with one sender antenna and one receiver antenna to MIMO-OFDM. Zelst and Schenk in source[9] with considering all the necessary changes in synchronization algorithm, channel estimation, ..., have generalized the OFDM based standard of IEEE-802.11a to MIMO.

The proposed synchronizing algorithms for OFDM based systems are categorized to the following two main groups [10]:

A. Before FFT algorithms

The above-mentioned algorithms are divided to two groups of input based algorithms and non input based algorithms as follows:

Non input based algorithms: this group of algorithms estimates the synchronization parameters using the special structure of OFDM symbols. This group is also called cyclic prefix based methods [11] and

[12]. Input based algorithms: this group of algorithms uses the educational symbols sent in information frames to estimate synchronization parameters [13], [14], [15] and [8].

B. After FFT algorithms

The algorithms of this group are also categorized in two groups of pilot based algorithms and direct decision algorithms. In comparing the two algorithms, before FFT algorithms are faster than after FFT algorithms, but after FFT algorithms has a higher throughput spectral.

III. MAXIMUM LIKELIHOOD ESTIMATION (MLE) ALGORITHM

The algorithm generates extremely accurate estimates even when the offset is great to demodulate the data values. The estimation error is insensitive to channel spreading and frequency selective fading [16].

The transmitter sends $X(K)$ data for $K=0, \dots, N-1$:

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn} ; n = 0, \dots, N - 1 \quad (1)$$

Adding the cyclic prefix(cp) and considering carrier frequency offset estimation, the $r(n)$ vector is formed as following:

$$r^{cp}(n) = e^{j2\pi\Delta f n} \times s^{cp}(n) * h(n) + Z(n); n = 0, \dots, N_s - 1 \quad (2)$$

Finally, we remove cp:

$$r(n) = e^{j2\pi\Delta f n} s^{cp}(n) * h(n) + Z(n); n = Ncp, \dots, N_s - 1 \quad (3)$$

To estimate the carrier frequency offset of $r(n)$ vector, r_1 and r_2 vectors are formed as follow:

$$r_1(n) = e^{j2\pi\Delta f Ncp} e^{j2\pi\Delta f n} (s(n) * h(n)) ; n = 0, \dots, \frac{N}{2} - 1 \quad (4)$$

And

$$r_2(\hat{n}) = e^{j2\pi\Delta f Ncp} e^{j2\pi\Delta f \hat{n}} (s(\hat{n}) * h(\hat{n})) ; \hat{n} = \frac{N}{2}, \dots, N - 1 \quad (5)$$

Assuming $d(n)=s(n)*h(n)$ and performing some calculations we have:

$$r_2^* \cdot r_1 = e^{j2\pi\Delta f \left(\frac{N}{2}\right)} |d(n)|^2 \quad (6)$$

$$\Gamma = \frac{r_2^* \cdot r_1}{|r_2^* \cdot r_1|} = e^{j2\pi\Delta f \left(\frac{N}{2}\right)} = e^{-j2\pi\Delta f N} = \cos \pi N \Delta f - j \sin \pi N \Delta f \quad (7)$$

Finally the carrier frequency offset estimation is performed as follow:

$$-\tan(\pi N \Delta f T) = \frac{Im(\Gamma)}{Re(\Gamma)} \Rightarrow \widehat{\Delta f} = \frac{1}{\pi} \tan^{-1} \left(-\frac{Im(\Gamma)}{Re(\Gamma)} \right) \quad (8)$$

IV. CHANNEL'S COEFFICIENTS ESTIMATION ALGORITHMS

The OFDM system based on pilot channel estimation is given in Figure 2. The binary information is first grouped and mapped according to the modulation in "signal mapper". After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length N $\{X(k)\}$ into time domain signal $\{x(n)\}$ with the following equation:

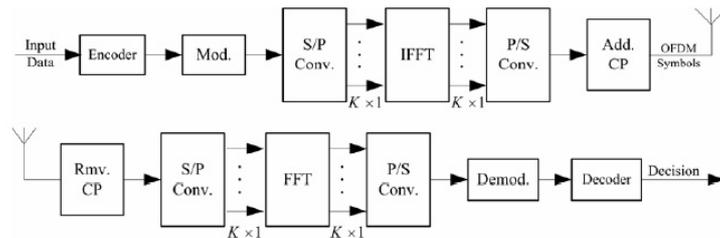


Figure 2. Displaying a OFDM system.

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn} ; n = 0, \dots, N - 1 \quad (9)$$

where N is the DFT length.

Following IDFT block, guard time (cyclic prefix(cp)), which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-carrier interference (ICI). The resultant OFDM symbol is given as follows:

$$s^{cp}(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k) e^{j\frac{2\pi}{N}k(n-Ncp)} ;$$

$$n = 0, \dots, N + Ncp - 1 \quad (10)$$

Then vector r rate in the presence of carrier frequency offset is:

$$r^{cp}(n) = e^{j2\pi\Delta f n} \times s^{cp}(n) * h(n) + Z(n); n = 0, \dots, N_s - 1 \quad (11)$$

With eliminating cp and doing a series of operations ,we have:

$$r(n) = e^{j2\pi\Delta f(n+Ncp)} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(K) \sum_{L=0}^{L-1} h_L e^{-j\frac{2\pi}{N}kL} e^{j\frac{2\pi}{N}kn} + Z(n) \quad (12)$$

Where Z (n) is White Gaussian Noise with an average of 0 . The output of the receiver is as follows:

$$y(n) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(n) e^{-j\frac{2\pi}{N}kn} ; k = 0, \dots, N - 1 \quad (13)$$

Result

$$y(n) = e^{j2\pi\Delta f Ncp} \sum_{i=0}^{N-1} x(i) H(i) \delta_{i,k} + Z(k) \quad (14)$$

And

$$\delta_{i,k} = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(CFO+i-k)} = sinc(CFO + i - k) e^{j\pi(CFO+i-k)}; i, k = 0, \dots, N - 1 \quad (15)$$

4.1. LS Channel Estimation

Channel information is required at receiver for signal detection. However, There are different methods of channel estimation such as pilot aided (Li, 2002) and blind (Gao and Nallanathan, 2007) approaches, the first method is chosen as a channel estimation method in this study due to its less complexity. According to sampling theory (Oppenheim and Schafer, 1999), Pilots are inserted equal-spaced among subcarriers in frequency domain at transmitter, which are known at receiver and will be extracted to estimate channel at pilot subcarriers and interpolation is implemented for channel estimation in another subcarriers. In the analysis, channel is estimated with LS (Coleri *et al.*, 2002) method at pilots, then linear interpolation is used to complete the estimation (Coleri *et al.*, 2002; Hsieh and Wei, 1998). The LS estimate of the channel can be obtained as

$$\begin{bmatrix} \hat{H}(k_0) \\ \hat{H}(k_1) \\ \vdots \\ \hat{H}(k_{L-1}) \end{bmatrix}_{L \times 1} = \begin{bmatrix} 1 & e^{-j\frac{2\pi}{N}k_0} & e^{-j\frac{2\pi}{N}2k_0} & \dots & e^{-j\frac{2\pi}{N}k_0(L-1)} \\ 1 & e^{-j\frac{2\pi}{N}k_1} & e^{-j\frac{2\pi}{N}2k_1} & \dots & e^{-j\frac{2\pi}{N}k_1(L-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi}{N}k_{L-1}} & e^{-j\frac{2\pi}{N}2k_{L-1}} & \dots & e^{-j\frac{2\pi}{N}k_{L-1}(L-1)} \end{bmatrix}_{L \times L} \times \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix}_{L \times 1} \quad (16)$$

And

$$\underline{H}_{L \times 1} = [WL]_{L \times L} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix}_{L \times 1} \quad (17)$$

$$\begin{aligned} \hat{h}_{L \times 1} &= \underline{W} \underline{L}^{-1} \underline{H}_{L \times 1} & (18) \\ \Rightarrow \hat{h}_{L \times 1} &= \underline{W} \underline{L}^{-1} \underline{y}(k)_{L \times 1} \text{ if: } k = \text{pilot} & (19) \end{aligned}$$

The channel response at the k^{th} sub-carrier estimated from the previous symbol $\{H_e(k)\}$ is used to find the estimated transmitted signal $\{X_e(k)\}$.

$$\hat{x}(k) = \frac{y(k)}{\hat{H}(k)}, k = 0, 1, \dots, N-1, k \neq k_0, k_1, \dots, k_{L-1}$$

$$X(\tilde{k}): \tilde{k} \in \text{pilot} \Rightarrow H(\tilde{k}) \cong \frac{y(\tilde{k})}{x(\tilde{k})}; \tilde{k} = 0, \dots, L-1 \quad (20)$$

4.2.LMS Channel Estimation

There are different iterative algorithms which are used to improve channel estimation and various methods are obtained as initial estimation. Also estimation in each iteration, can be used as side information and feed back to system to achieve better result in next iteration. The necessary steps carried out in LMS channel estimation are given below:

- 1- Initially the channel is estimated by using LSE technique, giving $\hat{H}_{LS}[n]$.
- 2- After finding the coefficients, the estimation of the channel becomes

$$\hat{H}_{LMS}[n] = \hat{W}^H[n] \hat{H}_{LS}[n] \quad (21)$$

Where

$$\hat{H}_{LS}[n] = [\hat{H}_{LS}[n] \hat{H}_{LS}[n-1] \dots \hat{H}_{LS}[n-1+M]] \quad (22)$$

Where M is LMS filter length

- 3- Error at iteration n is given by

$$E[n] = \hat{H}_{LS}[n] - \hat{H}_{LMS}[n] \quad (23)$$

- 4- Co-efficient are updated according to

$$\hat{W}[n+1] = \hat{W}[n] + \mu \hat{H}_{LS}[n] E^*[n] \quad (24)$$

Where μ is the adjustable step-size parameter.

- 5- Error given by weight vector is

$$e[n] = W[n] - \hat{W}[n] \quad (25)$$

Mean Square Error (MSE) given by the LMS algorithm is defined as $D[n] = \text{Tr}[K(n)]$ Where $K(N) = E[e(n) e^*(n)]$ And $E[.]$ shows the expectation operator. For real-time wireless communication, the value of the step-size parameter is taken very small. For slow co-efficient updating with better

performance $\mu = 0$ is used but for less computational time $\mu = 1$ is used, giving $\hat{H}_{LMS}[n + 1] = \hat{H}_{LMS}[n]$

As illustrated in Fig. 3, LMS algorithm is applied to receiver and the channel which was estimated in each iteration is used for next iteration, additionally the output signal is fed to source signal for next channel estimation. Another important factor in channel estimation through this method is μ which influences on estimation and should be precisely chosen[17].

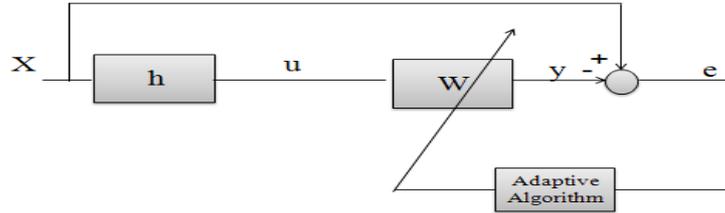


Figure 3. Implementing LMS algorithm in receiver.

In this method the coefficients of the vector H_n are obtained applying LMS recurrence as following:

$$\hat{H}_n = \hat{H}_{n-1} - \mu \times e \times X^* \quad (26)$$

Where

n = The iteration state, e = The signal error, μ = A coefficient between 0-1

V. SIMULATION RESULTS

The proposed algorithm is tested for a OFDM system. The blocksize of the OFDM symbol is kept at $N = 64$. The simulations are carried out at different SNR and in frequency selective faded environment. The frequency selective faded channel is implemented as a tapped delay line such that the delays are sample or T spaced. The assumed system has a QPSK modulation and L is the tap of channel.

5.1. Simulation results for LS algorithm

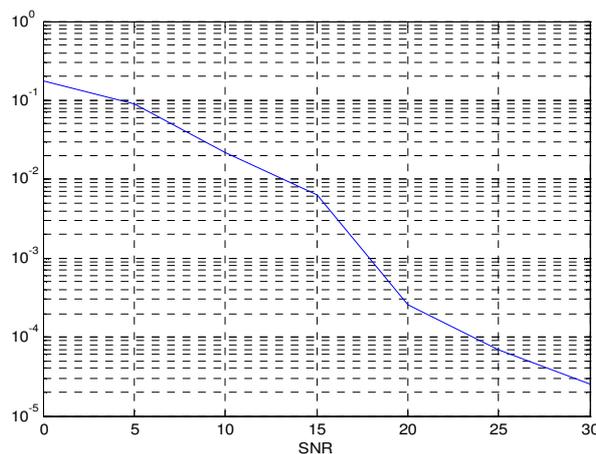


Figure 4. CFO estimation for SISO-OFDM systems with MLE algorithm, $L=5$.

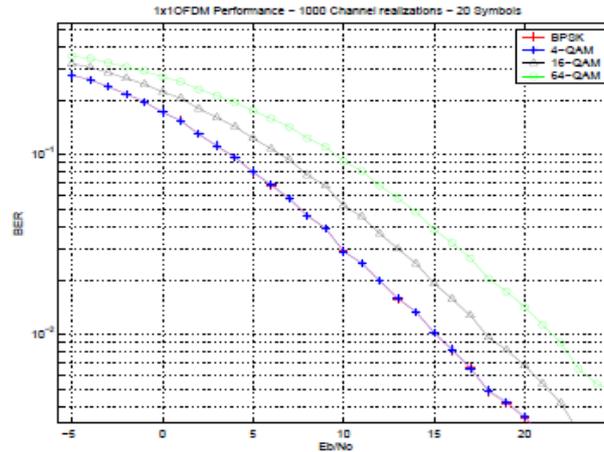


Figure 5. Bit error ratio performance of an OFDM system under frequency selective fading as a function of signal to noise ratio per bit.

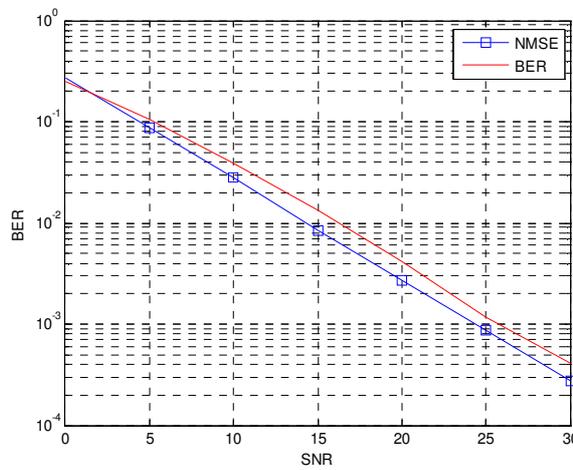


Figure 6. Channel estimation in SISO-OFDM systems $L=5$ without synchronization.

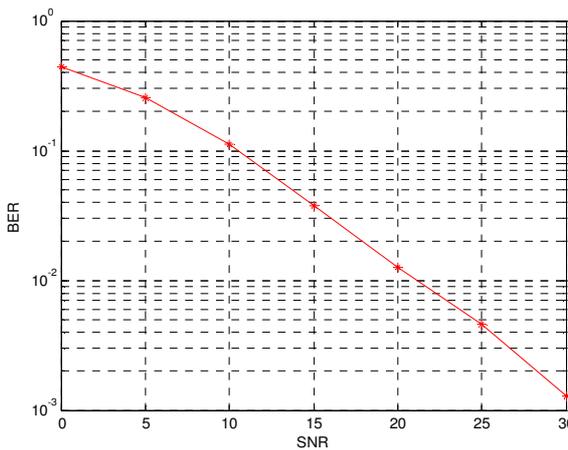


Figure 7. Channel estimation in SISO-OFDM systems $L=5$ with synchronization.

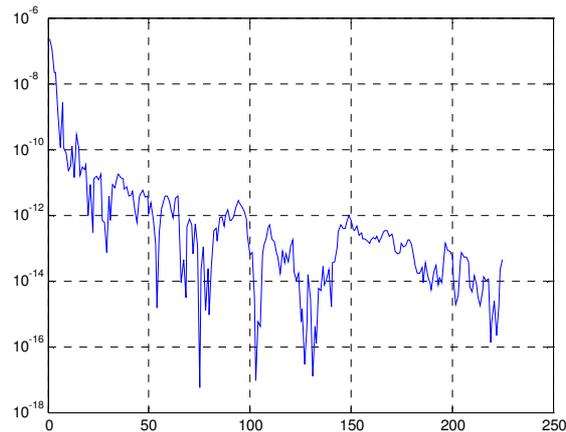


Figure 8. Joint channel estimation and synchronization for SISO-OFDM systems.

5.2.Simulation results for LMS algorithm

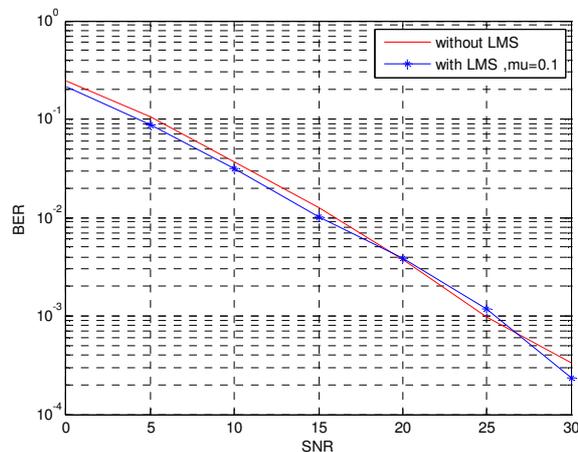


Figure 9. Channel estimation in SISO-OFDM systems L=5 without synchronization and LMS algorithm $\mu=0.1$.

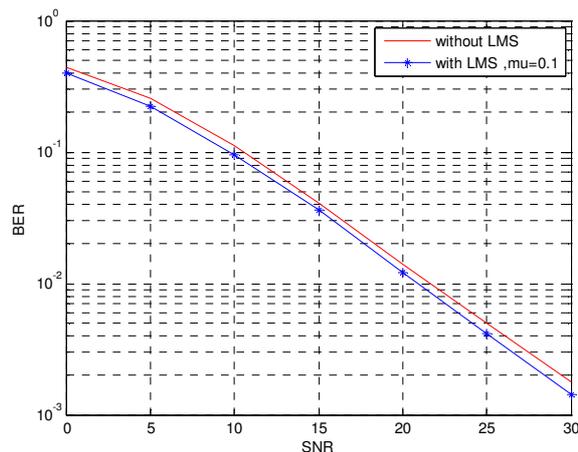


Figure 10. Channel estimation in SISO-OFDM systems L=5 with synchronization and LMS algorithm $\mu=0.1$.

Figure 4 shows CFO estimation in SISO-OFDM systems with MLE algorithm, L=5. Each of the figures 5 to 8 show channel estimation in SISO-OFDM systems with LS method. Figures 9, 10 show channel estimation in SISO-OFDM systems with LMS algorithm for $\mu=0.1$. Adding a LMS iterative algorithm to system, improves the channel estimation performance. Simulation results proved the

acceptable BER performance of iterative channel estimation algorithm, which is closed to the ideal channel.

VI. CONCLUSION

Estimation of channel coefficients and synchronization parameters are two main challenges in realization of OFDM-based systems which are practical. In almost all published references till now, estimation of channel coefficients is done with the assumption of total frequency synchronously of transmitter and receiver. The created frequency synchronously between transmitter and receiver, in practice, is always exposed to risk due to presence of factors such as Doppler phenomenon and phase noise. Therefore for exact estimation of fading channel status, it's necessary to keep the created frequency synchronously between transmitter and receiver, uninterrupted.

For improving accuracy of channel estimation, LMS iterative algorithm was added to receiver which includes a feedback of output and improves the BER performance of system, closed to the ideal channel performance.

Simulation results proved the acceptable BER performance of iterative channel estimation algorithm, which is closed to the ideal channel.

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