A GEOMETRIC PROGRAMMING BASED MODEL FOR COST MINIMIZATION OF TURNING PROCESS WITH EXPERIMENTAL VALIDATION

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ABSTRACT

The profit of any manufacturing unit is dependent on how much the firm can cut on its production cost. So, for increasing the profit, the production cost has to minimize. The production cost of turning process is dependent on the cutting parameters including the cutting speed, the feed rate and the depth of cut. If these parameters are set to the optimal level, then, the overall cost of the operation can be minimized. In this research paper, a geometric programming based approach to minimize the cost of the turning process is proposed. A mathematical model was developed depicting the production cost in terms of cutting parameters including cutting speed and feed rate with in some operating constraints. The results of the experimental validation of the model show that the proposed method provides a systematic, easy, effective and efficient technique to obtain the minimum production cost of turning process.

KEYWORDS: *Cost, Geometric Programming, Mathematical Model, Optimization*

I. INTRODUCTION

The selection of optimal cutting parameters, like the cutting speed, depth of cut and feed rate, forms a very important part of the cutting process. Turning is the operation performed most commonly in industries and manufacturing firms. Researchers have been trying from a long time to optimize the turning process by finding the optimal values of the turning process parameters like the cutting speed, feed rate and depth of cut that produces the optimum results. Generally, In Workshops, the cutting parameters are selected from machining databases or specialized handbooks, but the values obtained from these sources are actually the starting values and not the optimal values. Optimal cutting conditions are the key to economical cutting operations. Optimization of metal cutting operations means determination of the optimal set of operating conditions to satisfy an economic objective with in the operating constraints. The optimization is performed with respect to objective functions to satisfy the minimum production cost, maximum production rate, maximum profit rate or a desired combination of these three options [1, 2, 3]. Optimization of cutting parameters is generally a difficult task which involves the knowledge of machining, empirical equations of tool life, forces, powers, surface finish, etc. for development of an effective optimization criterion and good commands over the mathematical and numerical optimization techniques [4]. An analytical model for simultaneous determination of the optimal cutting conditions and the optimal tool replacement policy in the constrained cutting operation was investigated by using geometric programming [5]. In any optimization problem, it is very crucial to identify the prime objective called as the objective function or optimization criterion [6]. In manufacturing processes, the most commonly used objective function is the specific cost [7, 8].

The technology of metal cutting has improved significantly over time due to contributions from many branches of engineering with a common goal of achieving higher cutting process efficiency and reducing the overall cost of the process. Selection of optimal cutting conditions is a key factor for achieving this goal. In any metal cutting operation, the manufacturer seeks to set the process control variables at their optimal operating conditions with minimum level of variability in the outputs [9]. Walvekar and Lambert used geometric programming for the selection of machining variables. The optimum values of both cutting speed and feed rate were found out as a function of depth of cut in multi-pass turning operations [10]. Wu et. al analyzed the problem of optimum cutting parameters selection by finding out the optimal cutting speed which satisfies the basic manufacturing criterion [11]. Basically, this optimization procedure, whenever carried out, involves partial differentiation for the minimization of unit cost, maximization of production rate or maximization of profit rate. These manufacturing conditions are expressed as a function of cutting speed. Then, the optimum cutting speed is determined by equating the partial differentiation of the expressed function to zero. This is not an ideal approach to the problem of obtaining an economical metal cutting. The other cutting variables, particularly the feed rate also have an important effect on cutting economics. Therefore, it is necessary to optimize the cutting speed and feed rate simultaneously in order to obtain an economical metal cutting operation. The process of the metal cutting depends upon the features of tools, input work materials and machine parameter settings influencing process efficiency and output quality characteristics or responses. A significant improvement in process efficiency may be obtained by process parameters optimization that identifies and determines the regions of critical process control factors leading to desired outputs or responses with acceptable variations ensuring a lower cost of manufacturing [12]. Mathematical models can be very helpful in optimizing machining problems [13]. In the optimization of cutting parameters, several methods are used. Some are based on extensive experimentation which is quite laborious and lengthy process and its result may also vary in different conditions. Testing of materials like tool life test may require large amount of metals and considerable tool wear, so, it cannot be used for precious metals.

The aim of this research paper is the construction of a mathematical model describing the objective function in terms of the cutting parameters with some operating constraints, then; the mathematical model is optimized by using geometric programming approach. The developed model and program can be used to determine the optimal cutting parameters to satisfy the objective of obtaining minimum production cost of turning process under different operating constraints. The results of the mathematical model are obtained by using suitable software. This research paper proposes a very simple, effective and efficient way of optimizing the production cost of the turning process with some operating constraints such as the maximum cutting speed, maximum feed rate, power requirement, surface roughness. This paper also remarks the advantages of using geometric programming optimization approach over other optimization approaches.

II. MATHEMATICAL MODELING FOR COST MINIMIZATION

The unit cost for the total cost U to produce a part by turning operation can be expressed in terms of various costs as follows:

U = Machining Cost + Tooling Cost + Set-up Cost (1)

$$U = (C_m + C_0)t_m + \{(C_m + C_0 t_c) + C_t\}\frac{t_m}{T} + C_0 t_h \qquad (2)$$

The Taylor's tool life (T) used in Eq. (1) is given by:

$$T = \left(\frac{Z}{f^p\,v}\right)^{\frac{1}{n}} \qquad (3)$$

Where, n, p and Z depend on the many factors like tool geometry, tool material, work piece material, etc.

On substituting Eq. (3) in Eq. (2), we get

$$U(v,f) = C_{01}v^{-1}f^{-1} + C_{02}v^{\left(\frac{1}{n}\right)-1}f^{\left(\frac{p}{n}\right)-1} \qquad (4)$$

2.1 The machining constraints

There are many constraints which impose restrictions on the choice of the cutting parameters. These constraints arise due to various considerations like the maximum cutting speed, maximum feed rate, power limitations, surface finish, surface roughness, etc.

2.1.1 Maximum cutting speed

The increasing of cutting speed also increases the tool wear, therefore, the cutting speed has to be kept below a certain limit called the maximum cutting speed.

$$v \leq v_{max.} \tag{5}$$

$$C_{11} v \leq 1 \tag{6}$$

Where $C_{11} = \frac{1}{v_{max.}}$

By using the method of primal and dual programming of geometric programming, the maximum value of dual function or the minimum value of primal function is given by:

$$v(\lambda) = \left[\frac{C_{01}}{\lambda_{01}}(\lambda_{01} + \lambda_{02})\right]^{\lambda_{01}} \left[\frac{C_{02}}{\lambda_{02}}(\lambda_{01} + \lambda_{02})\right]^{\lambda_{02}} \left[\frac{C_{11}}{\lambda_{11}}\right]^{\lambda_{11}} \tag{7}$$

Subject to the following constraints:

$$\lambda_{01} + \lambda_{02} = 1 \tag{8}$$

$$-\lambda_{01} + \left\{\left(\frac{1}{n}\right) - 1\right\}\lambda_{02} + \lambda_{11} = 0 \tag{9}$$

$$-\lambda_{01} + \left\{\left(\frac{p}{n}\right) - 1\right\}\lambda_{02} = 0 \tag{10}$$

And the non-negativity constraints are:

$$\lambda_{01} \geq 0, \lambda_{02} \geq 0 \text{ and } \lambda_{11} \geq 0 \tag{11}$$

On adding Eq. (8) and Eq. (10), we get

$$\lambda_{02} = n \tag{12}$$

From Eq. (8) and Eq. (12), we get

$$\lambda_{01} = 1 - n \tag{13}$$

From Eq. (9), (12) and (13), we get

$$\lambda_{11} = 1 - p \tag{14}$$

Therefore, the maximum value of dual function or the minimum value of primal function is given by:

$$v(\lambda) = \left(\frac{C_{01}}{1-n}\right)^{1-n} \left(\frac{C_{02}}{n}\right)^{n} \{C_{11}(1-p)\}^{1-p} \tag{15}$$

Now, $\lambda_{11} = \frac{C_{11} v}{v(\lambda)}$ (16)

Therefore, from (15) and (16), we get

$$v = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \tag{17}$$

And, $\lambda_{01} = \frac{C_{01} v^{-1} f^{-1}}{v(\lambda)}$ (18)

Therefore from (17) and (18), we get

$$f = \frac{(C_{01} \times C_{11})}{(1-n) \times \lambda_1 \times \left(\frac{C_{01}}{1-n}\right)^{1-n} \left(\frac{C_{02}}{n}\right)^{n} (C_{11})^{1-p}} \tag{19}$$

2.1.2 Maximum feed rate

In rough machining operations, feed rate is taken as a constraint to achieve the maximum production rate.

$$f \leq f_{max.} \tag{20}$$

$$C_{11} f \leq 1 \tag{21}$$

Where $C_{11} = \frac{1}{f_{max.}}$

Following the same procedure as described in 2.1.1, we get the following values:

$$\lambda_{01} = 1 - n \tag{22}$$

$$\lambda_{02} = n \tag{23}$$

$$\lambda_{11} = 1 - p \tag{24}$$

$$f = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \tag{25}$$

$$v = \frac{(C_{01} \times C_{11})}{(1-n) \times \lambda_1 \times \left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \tag{26}$$

2.1.3 Power constraint

The maximum power available for the turning operation will be a constraint in the turning operation, which has to be taken in to consideration. The power available for the turning operation is given by:

$$P = \frac{F \times v}{6120\eta} \leq P_{max.} \tag{27}$$

$$C_{11} v \leq 1 \tag{28}$$

Where $C_{11} = \frac{F}{6120\eta P_{max.}}$

Following the same procedure as described in 2.1.1, we get the following values:

$$\lambda_{01} = 1 - n \tag{29}$$

$$\lambda_{02} = n \tag{30}$$

$$\lambda_{11} = 1 - p \tag{31}$$

$$v = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \tag{32}$$

$$f = \frac{(C_{01} \times C_{11})}{(1-n) \times \lambda_1 \times \left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \tag{33}$$

2.1.4 Surface roughness

Surface roughness can be used as a constraint in finishing operations. Therefore, it becomes a very important factor in determining finish cutting conditions. Surface roughness can be expressed in terms of feed as follows:

$$R_a = \frac{f^2}{32R} \tag{34}$$

$$R_a C_{11} \leq \frac{f^2}{32R} \tag{35}$$

Following the same procedure as described in 2.1.1, we get the following values:

$$\lambda_{01} = 1 - n \tag{36}$$

$$\lambda_{02} = n \tag{37}$$

$$\lambda_{11} = \frac{(1-p)}{2} \tag{38}$$

$$v = \frac{\left[\frac{(1-p)}{2}\right]\left[\frac{C_{01}}{1-n}\right]^{1-n}\left[\frac{C_{02}}{n}\right]^{n}[C_{11}]^{1-p}}{C_{11}} \tag{39}$$

$$f = \frac{C_{01} \times C_{11}}{(1-n)\left\{\frac{(1-p)}{2}\right\}\left\{\left[\frac{C_{01}}{1-n}\right]^{1-n}\left[\frac{C_{02}}{n}\right]^{n}[C_{11}]^{1-p}\right\}^{2}} \tag{40}$$

2.2 Experimental validation of the mathematical model

For validation of the mathematical model, experimental values were used from [14] and [15]. The values of the various parameters used in the experimental validation are as follows:

a = b = p = 1

n = 0.9

C_0 = 0.5 \$/min.

C_m= 0.2 \$/piece

C_t= 2.5\$/edge.

d = 50 mm.

l = 300 mm

R= 1.2 mm.

R_a = 10 µm.

t_c = 0.5 min.

t_h = 0.5 min.

η = 0.85.

Z=10

Table 1: Experimental values of cutting speed (v) and feed rate (f):

S. No.	Cutting speed (v) m/min.	Feed rate (f) mm./rev.
1	105	0.02
2	115	0.04
3	125	0.06
4	135	0.08
5	145	0.10
6	155	0.12
7	165	0.14
8	175	0.16
9	185	0.18
10	195	0.20

2.2.1 Validation for production cost model of turning process:

Table 2: Values of constants for production cost model:

S. No.	Parameter	Formulae	Value
1	C_{01}	C_{01}= constant = $(C_0 + C_m)\left(\frac{\pi dl}{1000}\right)$	33
2	C_{02}	C_{02} = constant = $(t_c C_0 + C_t)\left(\frac{\pi dl}{1000Z^{\frac{1}{n}}}\right)$	13

Table 3: Variation of the production cost (U) versus cutting speed (v) and feed rate (f):

S. No.	Cutting speed (v) m/min.	Feed rate (f) mm./rev.	Production Cost (U) \$/piece
1	105	0.02	29.819
2	115	0.04	22.550
3	125	0.06	20.625
4	135	0.08	19.945
5	145	0.10	19.721
6	155	0.12	19.704
7	165	0.14	19.791
8	175	0.16	19.934

9	185	0.18	20.107
10	195	0.20	20.297

Table 4: Optimum cutting parameters for minimum production cost:

S.No.	Optimum cutting speed (v) (m/min.)	Optimum feed rate (f) (mm. /rev.)	Optimum production cost (U) ($/piece)
1	155	0.12	20

III. Results and Discussion

3.1 Figures:

The figures obtained from the implementation of the mathematical model are as follows:

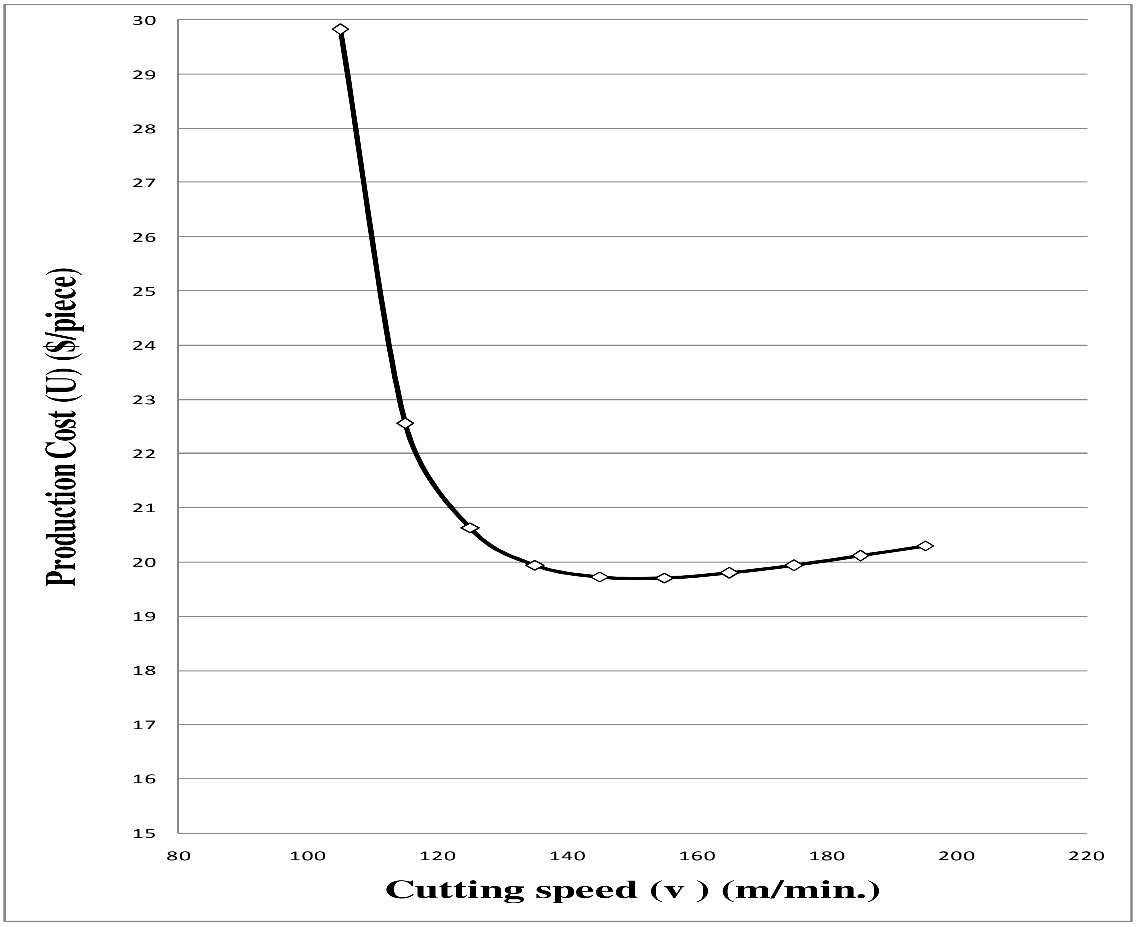

Figure 1: Variation of production cost versus cutting speed

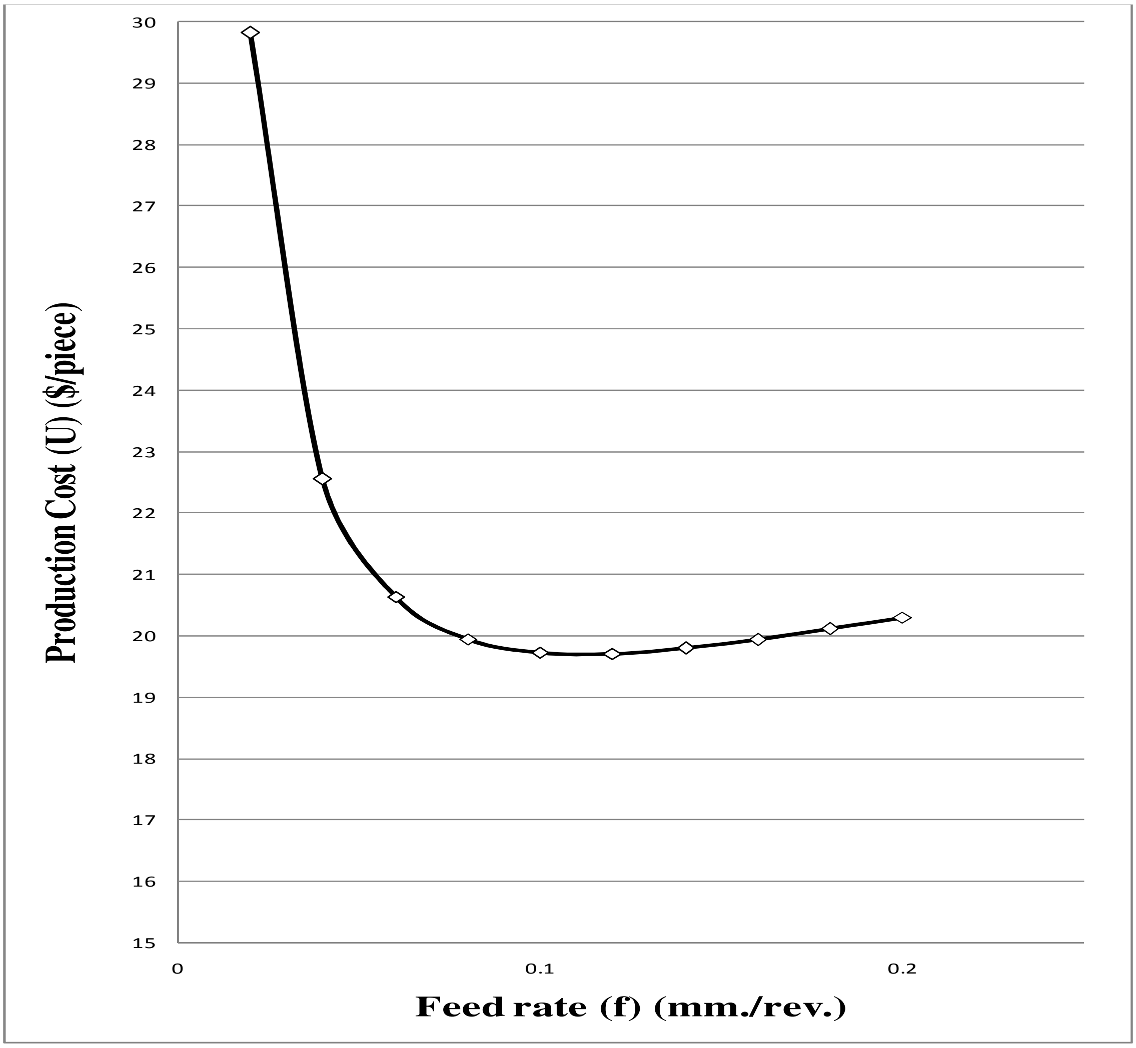

Figure 2: Variation of production cost versus feed rate

3.2 Discussion of results:

It is evident from the curves obtained between production cost and cutting speed that a smaller value of cutting speed results in a high production cost. It is due to the fact that a smaller cutting speed increases the production time of parts and the associated costs with it. Also, it will decrease the profit due to the production of a lesser number of parts. However, if the cutting speed is too high, it will also result in a high production cost due to excessive tool wear and the increased downtime. The optimum cutting speed is somewhere between "too slow" and "too fast" which will yield the minimum production cost.

The curves between the production cost and the feed rate reveal that a smaller feed rate will result in high production cost. A lesser feed rate means the number of revolutions should be increased. The more the number of revolutions, the more will be the production time and the associated tool wear. Even a very high feed rate is not advisable as it will increase the tool wear and surface roughness resulting in increased machining costs. So, the optimum feed rate is somewhere between "too small" and "too high" which will result in the minimum production cost. The overall results can be stated as under:

1. The obtained models can be used for finding the optimal range of cutting parameters in turning operation.

2. Obtained models can be used to determine the optimum cutting speed and feed rate which will satisfy the objective of minimum production cost of turning operations for different cutting tool and work piece combinations.

3. Obtained model saves a considerable solution time in finding the optimum cutting parameters. It can prove to be of great help to manufacturing firms who want the optimal range of cutting parameters in minimum time to gain a competitive edge in the market by producing a quality product as quickly as possible and launch it in the market. There, quick solution is of more significance than an absolutely accurate result obtained after consumption of lot of time.
4. It has been shown that geometric programming approach can also be applied effectively and efficiently to optimize the turning operation.

IV. CONCLUSION

In this research paper, the cutting speed and feed rate were modelled for the minimum production cost of a turning operation. The maximum cutting speed, the maximum feed rate, maximum power available and the surface roughness was taken as constraints. The results of the experimental validation of the mathematical model reveal that the proposed method provides a systematic and efficient methodology to obtain the minimum production cost for turning. The mathematical model can be used to find the optimal values of the cutting speed and feed rate that will yield minimum production cost of turning process. It has been shown that the method of geometric programming can be applied successfully to optimize the production cost of turning process. Future scope of this work will be developing similar models to optimize other machining operations like milling and grinding. The effect of various tool parameters like the rake angle, material of the tool and work piece on the optimization problem can also be incorporated.

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NOMENCLATURE

C_{01}= constant = $(C_0 + C_m)\left(\frac{\pi dl}{1000}\right)$

C_{02} = constant = $(t_c C_0 + C_t)\left(\frac{\pi dl}{1000 Z^{\frac{1}{n}}}\right)$

C_{11}= constant

C_0 = machine cost per unit time (\$/min.)

C_m = machining cost per piece (\$/piece)

C_t = tool cost (\$/cutting edge)

d = diameter of the work piece (mm.)

f = feed rate (mm/revolution)

F = cutting Force (N)

l = length of the work piece (mm.)

n, p and Z are constants.

R= nose radius of the tool (mm)

R_a =average surface roughness (μm)

t_c = tool changing time (min.)

t_h = tool handling time (min.)

t_m= time required to machine a work piece = $\frac{\pi dl}{1000 vf}$ (min.)

T = tool life (min.)

U = total cost to produce a part by turning operation (\$/piece)

v = the cutting speed (m/min.)

η = efficiency of cutting

$\lambda_{01}, \lambda_{02}$ *and* λ_{11} are Lagrange multipliers.

Author

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