

IMPROVING THE ACCURACY OF RESIDUAL IONOSPHERIC ERRORS IN THE GPS NAVIGATION SYSTEM

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ABSTRACT

To improving accuracy of positioning in the GPS navigation system we need to eliminate ionospheric errors or at least reduce their effect. In this research we are going to introduce the suggested solutions to improving the accuracy of residual ionospheric errors. Create new ionospheric mathematical model without neglect higher orders that helps to eliminate ionospheric errors comparing to others models and making evaluations between all orders.

I. INTRODUCTION

There are two Techniques to improving or eliminate ionospheric errors. First Technique use (Combining Pseudorange) both broadcast frequency L1&L2 signals and the inverse square to measure delay directly by difference code measurements on each frequency, Although this technique may be cancelled 99.9% or eliminate ionospheric errors but it cannot use by users because it used only by U.S military, this technique used L1 and L2 carrier frequency to eliminate errors.

Second technique uses the ionospheric mitigation models. In this research we are going to use second technique, there are a number of different mitigation models that used to eliminating ionospheric errors such as: GRAPHIC algorithm, the Global Ionospheric Maps (GIMs), and the Klobuchar model.

II. IONOSPHERIC MODELS

GRAPHIC algorithm its method methodology to eliminate errors due to the code and advances the carrier phase measurements by taking the simple average of the code and carrier phase delay observables, upon adding and averaging the code and carrier phase range, the combined GRAPHIC measurement (ignoring the higher-orders of ionospheric error terms).

Global Ionospheric Maps (GIMs) provides an instantaneous "snapshots" of the global TEC distribution by interpolating, in both (space and time), the 6-8 simultaneous TEC measurements obtained from each GPS receiver every 30 sec.

The maps can be produced unattended in a real-time mode, with an update rate of 5-15 minutes, these are many products that contributed to the International Global Navigation Satellite System Services (IGS) Ionosphere working group in order to generate the combined IGS final and rapid GIMs.

Klobuchar model the last ionospheric mitigation technique called Klobuchar model which is the simplest and most widely used method to correct the ionospheric error. Klobuchar model together with the eight ionospheric coefficients broadcast as part of the navigation message and its parameters are updated at least once every six days. This algorithm can be used in real-time and it was designed to provide a correction for approximately 50 % Root Mean Square (RMS) of the ionospheric range delay.

III. MATHEMATICAL MODEL ANALYSIS

The equation of total range of errors that used in any model by GPS can be represented as:

$$P = (\sigma) + C (dt_{sat} + dt_{res}) + \Sigma UERE$$

Where:

P: Total range of errors measured by GPS

σ : Geometric range

UERE: User Equivalent Range Errors

($dt_{sat} + dt_{res}$): Clock offset

The part of user equivalent range errors (UERE) can be represented in some summation terms as:

$$\Sigma UERE = (Rel) + (Tro) + (Ion) + (K^{sat}) + (K_{res}) + (\epsilon)$$

Where:

Rel: Relativistic effect from satellite to receiver.

Tro: Tropospheric delay errors from satellite to receiver.

Ion: Ionospheric delay errors from satellite to receiver.

Ksat, Kres: Instrumentals delay.

ϵ : Noise.

From previous analysis we can see an ionospheric range error appears in the part of user equivalent range errors (UERE).

We used from this part, ionospheric error to find residual ionospheric errors in three cases (first, second and third orders).

3.1 Estimating Ionospheric Mathematical Model

To estimating ionosphere residual errors in the different orders, must be estimating equation of ionospheric errors in the general form.

Brunner & Gaussian (1991), the general term of refractive index of the ionosphere expressed:

$$\eta_{ion} = 1 - \left(\frac{C_x N_e}{2 f^2} \right) - \left(\frac{C_x C_y N_e (H_0 \cos\theta)}{2 f^3} \right) - (C_x^2 N_e^2 / 8 F^4) \quad (1)$$

Where:

$$C_x = \frac{e^2}{4 \pi^2 \epsilon_0 m_e}$$

$$C_y = \frac{\mu_0 e}{2 \pi m_e}$$

N_e : The electron density,

F : The frequency of the propagating signal,

H_0 : The strength of the magnetic field is the angle between the direction of the propagation of the electromagnetic wave and the vector of the magnetic field.

e : The electron charge ($-1.602 \times 10^{-19}C$).

ϵ_0 : The vacuum permittivity ($1/36\pi \times 10^{-9} F/m$).

m_e : The mass of an electron ($9.109 \times 10^{-31} kg$),

μ_0 : The vacuum permeability ($4\pi \times 10^{-7} H/m$).

$$\eta_g = \eta_p + f \frac{d \eta_p}{df} \quad (2)$$

$$\eta_p = \eta_{ion} = 1 + \frac{a_1}{f^2} + \frac{a_2}{f^3} + \frac{a_3}{f^4} \quad (3)$$

Substituting equation (2) in (3), then we find:

$$\eta_g = 1 - \frac{a_1}{f^2} - \frac{2 a_2}{f^3} - \frac{3 a_3}{f^4} \quad (4)$$

Where:

η_g : The refractive index of GPS signals in the vacuum.

η_p : The refractive index of GPS signals in the medium.

f : Carrier frequency used in GPS.

a_1, a_2, a_3 : Coefficients.

When the GPS signals penetrate through the ionosphere, the distance of the transmission paths due to the variation of the refractive index

$$\delta_g = e_{ion} = \int (\eta_g - 1) dl \quad (5)$$

Where:

e_{ion} : Ionospheric delay errors.

3.2 Estimating Ionosphere Different Orders

From equation (4) we can find terms of higher orders used to calculating effect of refractive index in the vacuum and ionosphere layers.

The different higher orders can show as:

$$- \int \left(\frac{a_1}{f_2}\right) dl \quad : \text{The value of first order}$$

$$- 2 \int \left(\frac{a_2}{f_3}\right) dl \quad : \text{The value of second order}$$

$$- 3 \int \left(\frac{a_3}{f_4}\right) dl \quad : \text{The value of third order}$$

The older researches shown ionospheric mathematical model by using first order only and neglected higher orders, in this paper we are going to estimating ionospheric mathematical model by using all orders, without neglect any orders. There are three cases: First case will neglect second and third orders and using first order only. Second case will neglect third order and using first and second order only. Third case using all orders without neglect any orders.

Substituting equations (4) in (5) then we find residual ionospheric errors in different cases shown in **Table1**.

Table1 Maximum vertical residual ionospheric error equation [unit=m]

Cases	order	Residual Ionospheric Errors
Case (1)	First	$40.3 \times F^{-2} \times \text{TEC}$
Case (2)	Second	$- 2.82 \times 106 \times F^{-3} \times \text{TEC}$
Case (3)	Third	$- 1.6 \times 103 \times F^{-4} \times N_m \times \text{TEC}$

IV. NUMERICAL RESULTS

In this part will show effects of orders to residual ionospheric errors in different parts such as:

4.1 Effect Of Orders On L1 And L2 At Constant Tec

Calculation of carrier frequency L1 and L2 to find residual ionospheric errors in all orders at constant total electron content equal $4.55 \times 10^{18} \text{ el/m}^2$ and $1.38 \times 10^{18} \text{ el/m}^2$ respectively **Table2**.

Table2 Maximum vertical residual ionospheric range error [unit=m]

TEC= $4.55 \times 10^{18} \text{ el/m}^2$ $N_m = 20 \times 10^{12} \text{ el/m}^3$			
Frequency	1st order effect($1/f^2$)	2nd order effect($1/f^3$)	3rd order effect($1/f^4$)
L1	4.350	0.7012	0.0195
L2	5.112	0.0831	0.0069

TEC= $1.38 \times 10^{18} \text{ el/m}^2$ $N_m = 6 \times 10^{12} \text{ el/m}^3$			
Frequency	1st order effect($1/f^2$)	2nd order effect($1/f^3$)	3rd order effect($1/f^4$)
L1	1.952	0.543	0.0121
L2	2.356	0.0289	0.0065

This calculation in **Table2** indicate to the different orders by using carrier frequency L1 and L2 at constant total electron content gives different reading of results, these results mean 2nd orders and 3rd

orders gives best results of residual ionospheric errors in both carrier frequency L1 and L2, 1st orders gives bad results. The reading of 3rd orders is very small in the range of millimeters, reading of 2nd orders in centimeters and reading of 1st orders in meters.

4.2 Effect of Orders on L1 at Different Tec

Show in the **Table3** calculation of carrier frequency L1 to find residual ionospheric errors in every order alone at different total electron content.

Table3 Maximum vertical residual ionospheric range error [unit=m] for every case alone, using L1 carrier frequency

Frequency L1=1575.24MHz			
TEC	1st order effect(1/f ²)	2nd order effect(1/f ³)	3rd order effect(1/f ⁴)
5×10 ¹⁶ el/m ²	1.62	0.811	0.0956
10×10 ¹⁶ el/m ²	1.95	0.791	0.0853
15×10 ¹⁶ el/m ²	2.34	0.766	0.0821
20×10 ¹⁶ el/m ²	2.56	0.725	0.0756
25×10 ¹⁶ el/m ²	2.61	0.678	0.0706
30×10 ¹⁶ el/m ²	3.16	0.642	0.0654
37×10 ¹⁶ el/m ²	3.91	0.601	0.0555
40×10 ¹⁶ el/m ²	4.21	0.583	0.0504
45×10 ¹⁶ el/m ²	4.53	0.552	0.0421
50×10 ¹⁶ el/m ²	4.92	0.503	0.0321

This results means the residual ionospheric errors of 2nd order in range centimeters so it cannot be neglect and residual ionospheric errors of 3rd order in range millimeters so it can be neglect because it not affecting in accuracy of GPS navigation system.

Show in the **Table4** and **Figure1** calculation of carrier frequency L1 to find total residual ionospheric errors in all cases at TEC.

Table4 Residual ionospheric error [unit=m], at different TEC [unit=m]

Frequency L1=1575.24MHz			
TEC	δ1 st order	δ1 st +2 nd order	δ1 st +2 nd +3 rd order
5×10 ¹⁶ el/m ²	1.62	0.809	0.7134
10×10 ¹⁶ el/m ²	1.95	1.159	1.0737
15×10 ¹⁶ el/m ²	2.34	1.574	1.4919
20×10 ¹⁶ el/m ²	2.56	1.835	1.7594
25×10 ¹⁶ el/m ²	2.61	1.932	1.8614
30×10 ¹⁶ el/m ²	3.16	2.518	2.4526
37×10 ¹⁶ el/m ²	3.91	3.309	3.2535
40×10 ¹⁶ el/m ²	4.21	3.627	3.5766
45×10 ¹⁶ el/m ²	4.53	3.978	3.9359
50×10 ¹⁶ el/m ²	4.92	4.417	4.3849

These results shown in **Table4** indicate to the using 2nd and 3rd orders we can have high accuracy, the accuracy of 3rd order is very small and near to accuracy of 2nd order so we can neglect calculation of 3rd order.

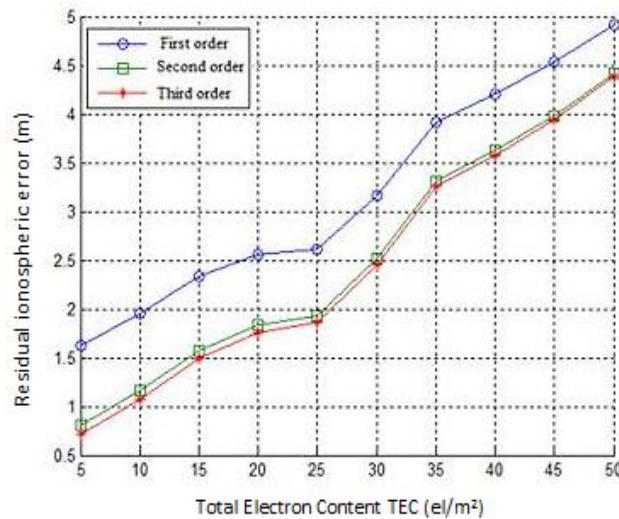


Fig.1 Residual ionospheric error [unit=m] at different TEC TEC [unit=m]

4.3 Effect of Orders on L1 at Different Elevation Angle

Show in the Table5 and Figure2 calculation of carrier frequency L1 to find total residual ionospheric errors in all cases at different elevation angle.

Table.5 Residual ionospheric error [unit=m], at different elevation angle

Frequency L1=1575.24MHz			
Elevation Angle	$\delta_{1^{st}}$ order	$\delta_{1^{st}+2^{nd}}$ order	$\delta_{1^{st}+2^{nd}+3^{rd}}$ order
0°	3.11	1.02	1.52
10°	2.67	0.95	1.34
20°	2.10	1.50	1.10
30°	1.84	2.31	2.01
40°	2.48	3.22	1.93
50°	2.85	2.46	1.20
60°	3.77	1.95	1.75
70°	3.13	2.02	2.91
80°	3.36	3.01	2.43
90°	4.60	3.42	2.01

These numerical results shown in Figure1 indicate to the value of 2nd and 3rd orders when using with 1st order gives minimum residual ionospheric errors that mean we will have good accuracy when using all orders together at different total electron content.

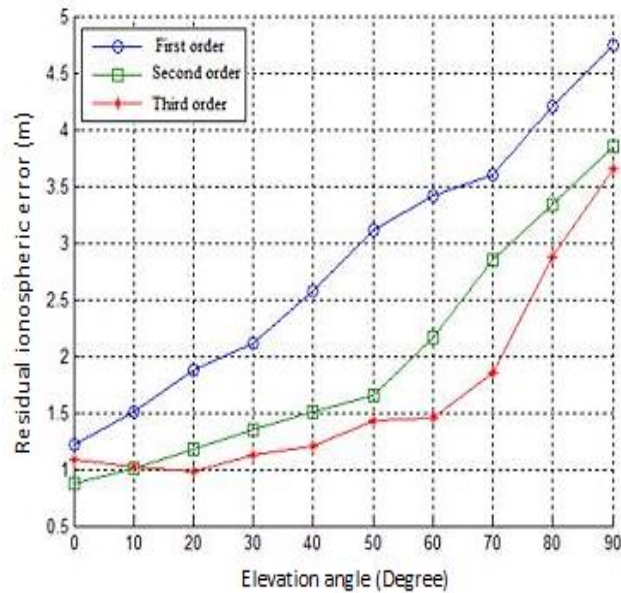


Fig.2 Residual ionospheric error [unit=m], at different elevation angle

These numerical results shown in **Figure 2** and **Table** indicate to the value of 2nd and 3rd orders when using with 1st order gives minimum residual ionospheric errors that mean we will have good accuracy when using all orders together at different elevation angle.

V. CONCLUSION

Create new ionospheric mathematical model without neglect higher orders that helps to eliminate ionospheric errors comparing to others models use first order only, The numerical results showed residual ionospheric errors on 2nd order in range centimeters so it cannot be neglected and residual ionospheric errors on 3rd order in range millimeters so it can be neglected, The numerical results showed that the 2nd and 3rd orders performance better than 1st order in residual ionospheric errors at different total electron content, The numerical results showed that the 2nd and 3rd orders performance better than 1st order in residual ionospheric errors at different elevation angle

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