A STUDY ON EXPLANATION OF TWO STAGE GAMBLING EXPERIMENT IN A NEW CONTEXT

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ABSTRACT

Amos Tversky and Eldar Shafir experimentally observed the violation of principle of sure thing in case of two stage gambling experiment. The classical decision theory is unable to explain this paradoxical departure in view of the logical statement of principle of sure thing. Such type of paradoxical departure of human behaviour in decision making can be explained using probabilistic mathematical frame work of quantum mechanics. The basic objective of this paper is to study the explanation of this departure using quantum probability interference of amplitudes.

KEYWORDS: *Principle of sure thing, Quantum Decision Theory, Interference of probability amplitudes, Two Stage Gambling Experiment.*

I. INTRODUCTION

In the early twentieth century physicists faced problem to explain certain physical phenomena using classical logic, probability and dynamics. Quantum mechanics was discovered to explain those experiments successfully. The mathematical basis of quantum mechanics discovered new type of logic, probability and dynamics called quantum logic, quantum probability and quantum dynamics respectively in order to study the behaviour of subatomic particles. Similarly the classical decision theory cannot explain some experiments related to human decision making. The principle of sure thing plays an important role in classical decision theory. The statement of the principle of sure thing is as follows: if X prefers Y, with the knowledge that the event Z happens and if X prefers Y knowing that the event Z does not happen then the happening or not happening of event Z does not affect the preference of X to Y. Other way one can say the happening or non happening of event Z does not affect the preference of X to Y. This principle becomes a basis of classical decision theory. But this principle contradicts with the results of some experiments related to psychology. People violate principle of sure thing in case of the two famous experiments namely two stage gambling experiment and the prisoner’s dilemma experiment. The classical decision theory fails to explain these experimental findings for over a decade. In recent years, a new trend based upon principles of quantum mechanics has come forward and has successfully explained many computationally hard problems by using quantum search algorithm[1,5], quantum factorization algorithm[4] and quantum cryptography [2,3].Hence we have enough evidence of the successful applications of quantum techniques in non-quantum mechanical problems. The perceptive and applicability of quantum principles to explain decision process have been discussed here. In particular the violation of principle of sure thing in case of two stage gambling experiment has been discussed using the interference effect of quantum probability amplitudes.

II. EXPERIMENTAL FINDINGS OF TEVERSKY AND SHAFIR

Amos Tversky and Eldar Shafir [6, 8] have significant contribution in the field of cognition and decision science. The experimental observations of two stage gambling problem conducted by Amos Tversky and Eldar Shafir [7] are as follows: In the first step of the game a coin is tossed and having 50% possibility of winning $\$200$ and 50% possibility of loosing $\$100$. The participants are allowed to play the same game again for the second time with or without knowing the outcome of the 1st game. As an observation of the experiment it is found that a maximum number of participants are prepared to play the game for second time after knowing the success or failure of the first game. But a very few are ready to play the game for a second time without knowing the result of the first game. In view of the principle of sure thing [9] all the participants should go for second game even if they know or don't know the outcome of the first game. Hence the principle of sure thing is violated in this experiment.

III. MATHEMATICAL NOTATIONS

The classical decision theory is unable to explain the contradiction of the experiment from the logical statement of sure thing principle. Such type of paradoxical departure of human behaviour in decision making can be modelled using probabilistic mathematical frame work of quantum mechanics. The successful explanation of the paradoxical departure was studied by a number of researchers [11, 14, 15, 16, 19] shows that the experimental behaviour of human being can be explained by the use of quantum mechanical principles. The mathematical frame work of quantum mechanics abstracted the state of human mind similar with quantum states in Hilbert space and the process of decision making formulated based on postulates of quantum mechanics [17,]. Yukalov and Sornette [10, 12, 13] have provided the detail theory of quantum decision science. They have explained postulates of quantum decision science similar to that of postulates of quantum mechanics. Khrennikov, Yukalov and Sornette [12, 13] have derived quantum interference of probability amplitudes that can explain various experimental findings of physiology. In this section various quantum mechanical notations related to two stage gambling experiment are discussed to represent the experimental facts. The quantum states and operators respectively are as follows:

3.1. Description Of Various Probabilities

Suppose $X_1 \rightarrow$ Success in first game and $A_1 \rightarrow$ Agree to play the game for second time. $X_2 \rightarrow$ Failure in first game and $A_2 \rightarrow$ Do not agree to play the game for second time.

Let $P(X_1)$ be the probability of success of the participants in the first game and $P(X_2)$ be the probability of failure of the participants in the first game.

Similarly $P(A_1|X_1) \rightarrow$ is the conditional probability of participants those agree to play the game for 2nd time with the knowledge of success in the 1st game.

$P(A_2|X_1) \rightarrow$ Is the conditional probability of participants those do not agree to play the game for 2nd time with the knowledge of success in the 1st game.

$P(A_1 X_1) \rightarrow$ Is the joint probability of success in 1st game and agrees to play the game for 2nd time.

$$P(A_1 X_1) = P(X_1)P(A_1|X_1) \quad (1(a))$$

$P(A_2X_1) \rightarrow$ Is the joint probability of failure in 1st game and agree to play the game for 2nd time.

$$P(A_2X_1) = P(X_1)P(A_2|X_1) \quad (1(b))$$

$P(A_1|X_2) \rightarrow$ Is the conditional probability of participants agreed to play the game for 2nd time with knowledge of failure in the 1st game.

$P(A_2|X_2) \rightarrow$ Is the conditional probability of participants those do not agree to play the game for 2nd time with knowledge of failure in the 1st game.

$P(A_1 X_2) \rightarrow$ Is the joint probability of failure in 1st game and agree to play the game for 2nd time.

$$P(A_1 X_2) = P(X_2)P(A_1|X_2) \quad (1(c))$$

$P(A_2X_2) \rightarrow$ Is the probability of failure in 1st game and do not agree to play the game for 2nd time.

$$P(A_2X_2) = P(X_2)P(A_2|X_2)$$

(1(d))

$P(A_1)$ → Is the probability of participants those agree to play the game for 2nd time without the knowledge of success or failure in the 1st game.

$P(A_2)$ → Is the probability of participants those do not agree to play the game for 2nd time without knowledge of success or failure in the 1st game.

From this we can conclude that the sum of participants agree to play the game for 2nd time and those who do not agree to play the game for 2^{nd} time without the knowledge of success or failure in the 1st game can be written as:

$$P(A_1) + P(A_2) = 1 \tag{1(e)}$$

3.2. Description Of Quantum States And Operators

Let $|A_1\rangle$ → be the state that corresponds to agree to play the game for second time with 100% probability. Similarly, $|X_1\rangle$ → be the state that corresponds success in first game with 100% probability, $|A_2\rangle$ → be the state that corresponds not agree to play the game for second time with 100% probability, $|X_2\rangle$ → be the state that corresponds success in first gamble with 0% probability, $|A_1X_1\rangle$ → be the state in which Probability of success in first game is 100% as well as agrees to play the game for second time is 100%, i.e.

$$|A_1X_1\rangle = |A_1\rangle|X_1\rangle$$

And $|A_2X_1\rangle$ → be the state in which Probability of success in first game is 100% as well as does not agree to play the game for second time 100%, i.e.

$$|A_2X_1\rangle = |A_2\rangle|X_1\rangle$$

Similarly let $|A_1X_2\rangle$ → be the state in which Probability of not success in first game is 100% as well as agrees to play the game for second time is 100%, i.e.

$$|A_1X_2\rangle = |A_1\rangle|X_2\rangle$$

And $|A_2X_2\rangle$ → be the state in which Probability of not success in first game is 100% as well as not agree to play second gamble is 100%, i.e.

$$|A_2X_2\rangle = |A_2\rangle|X_2\rangle$$

O_{SUC} → $Success$ operator which corresponds to the probability of success in first game.

O_{A_1} → Agree to play operator which corresponds to agree to play second game.

Let us define the operators corresponding to the Eigen states related to the experiment. Suppose $|A_1\rangle$ is an eigen state of the operator O_{A_1} with eigen value 1. In the same way $|A_2\rangle$ is an eigen state of the operator O_{A_1} with eigen value 0. Similarly $|X_1\rangle$ is an eigen state of the operator O_{SUC} whose eigen value is 1 and $|X_2\rangle$ is the eigen state of the operator O_{SUC} with eigen value 0. As we know eigen kets belong to different eigen values of a given operator are orthogonal to each other, so let us try to utilize these concepts in quantum decision model as follows:

$$O_{A_1}|A_1\rangle = |A_1\rangle,\ O_{A_1}|A_2\rangle = 0 \tag{2(a)}$$

$$O_{A_1}|A_1X_1\rangle = |A_1X_1\rangle,\ O_{SUC}|A_1X_1\rangle = |A_1X_1\rangle \tag{2(b)}$$

$$O_{SUC}|X_1\rangle = |X_1\rangle,\ O_{SUC}|X_2\rangle = 0 \tag{2(c)}$$

$$O_{A_1}|A_2X_1\rangle = 0,\ O_{SUC}|A_2X_1\rangle = |A_2X_1\rangle \tag{2(d)}$$

$$O_{A_1}|A_1X_2\rangle = |A_1X_2\rangle, O_{SUC}|A_1X_2\rangle = 0 \tag{2(e)}$$

$$O_{A_1}|A_2X_2\rangle = 0,\ O_{SUC}|A_2X_2\rangle = 0 \tag{2(f)}$$

$$\langle X_1|X_2\rangle = \langle X_2|X_1\rangle = 0,\ \langle A_1X_1|A_1X_2\rangle = \langle A_1X_2|A_1X_1\rangle = 0$$
$$\langle A_1|A_2\rangle = \langle A_2|A_1\rangle = 0,\ \langle A_2X_1|A_2X_2\rangle = \langle A_2X_2|A_2X_1\rangle = 0$$

IV. REPRESENTATION OF STATES IN SUCCESS-FAILURE SPACE

In the previous sections the basic operators and Eigen states related to the experiment have been discussed. Let us construct the ket representation of the state present in success-failure space. Let the state $|\Phi\rangle$ be the success/failure of the system as a linear combination.

$$|\Phi\rangle = \alpha_1|X_1\rangle + \alpha_2|X_2\rangle \tag{3}$$

The normalization condition satisfies $|\alpha_1|^2 + |\alpha_2|^2 = 1$.The success/failure state $|\Phi\rangle$ can be represented as linear combination of success state $|X_1\rangle$ and of failure state $|X_2\rangle$ with probability amplitude α_1 and α_2 respectively shown in the figure-1.
As O_{SUC} is the success operator so probability of success in first gamble can be calculated as :

$$P(X_1) = \langle\Phi|O_{SUC}|\Phi\rangle \tag{4}$$

From Equation (3) and Equation (2-c)

$$O_{SUC}|\Phi\rangle = O_{SUC}(\alpha_1|X_1\rangle + \alpha_2|X_2\rangle) = \alpha_1|X_1\rangle \tag{5}$$

The bra vector corresponding of Equation (3) will be:

$$\langle\Phi| = \alpha^*{}_1\langle X_1| + \alpha^*{}_2\langle X_2| \tag{6}$$

By combining Equations (4), (5), (6)

$$\begin{aligned} P(X_1) &= \langle\Phi|O_{SUC}|\Phi\rangle = [\alpha^*{}_1\langle X_1| + \alpha^*{}_2\langle X_2|\ ][\alpha_1|X_1\rangle] \\ &= \alpha^*{}_1\alpha_1\langle X_1||X_1\rangle + \alpha^*{}_2\alpha_1\langle X_2||X_1\rangle = \alpha^*{}_1\alpha_1 = |\alpha_1|^2 \end{aligned} \tag{7}$$

Hence

$$P(X_2) = 1 - P(X_1) = 1 - |\alpha_1|^2 = |\alpha_2|^2 \tag{8}$$

From Equations (7), (8) the success/failure is not an issue for decision by players in this experiment.

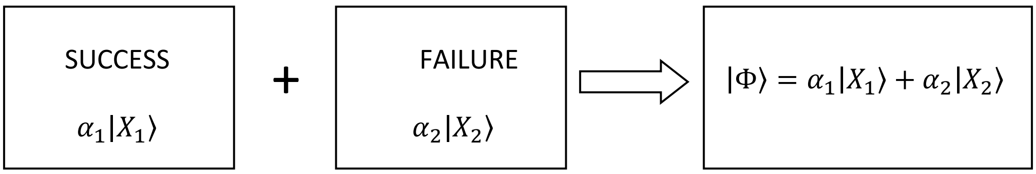

Figure-1

From the experiment it is observed that some agrees to play the game for second time while getting success in the first game and some does not agree to play the game for the second time. Hence three cases come in to picture:

CASE-1

Consider the case of players associated with “success in first game-agree to play the game for second time” and “success in first game –do not agree to play the game for second time” can be represented as a linear combination of $|A_1X_1\rangle$ and$|A_2X_1\rangle$. So the state of mind $|\Psi_1\rangle$ associated with success in the first game can be represented as a linear combination of $|A_1X_1\rangle$ (success-agree to play) and $|A_2X_1\rangle$ (success-not agree to play) is shown in the figure-2.

$$|\Psi_1\rangle = a_1|A_1X_1\rangle + b_1|A_2X_1\rangle \tag{9}$$

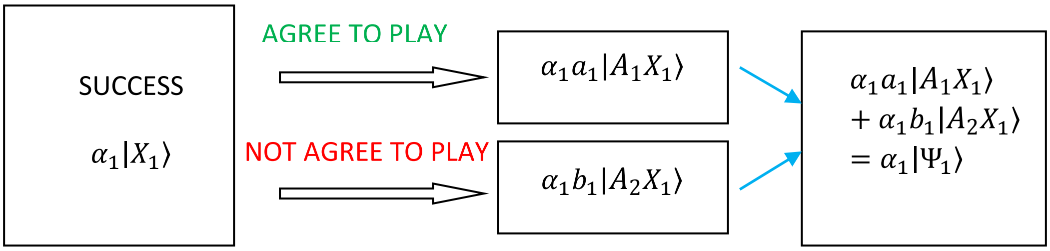

Figure -2

The normalization condition leads to $|a_1|^2 + |b_1|^2 = 1$. For computing success-agree to play we can use the state $|\Psi_1\rangle$ and operator O_{A_1} . Hence Probability of success in first game and agree to play the game for second time will be:

$$P(A_1|X_1) = \langle\Psi_1|O_{A_1}|\Psi_1\rangle \quad (10)$$

$$O_{A_1}|\Psi_1\rangle = a_1 O_{A_1}|A_1X_1\rangle + b_1 O_{A_1}|A_2X_1\rangle = a_1|A_1X_1\rangle \qquad (\because Equn - 2 - b, d)$$
$$\langle\Psi_1| = a^*{}_1\langle A_1X_1| + b^*{}_1\langle A_2X_1|$$
$$P(A_1|X_1) = \langle\Psi_1|O_{A_1}|\Psi_1\rangle = [a^*{}_1\langle A_1X_1| + b^*{}_1\langle A_2X_1|]a_1|A_1X_1\rangle$$
$$= a^*{}_1 a_1\langle A_1X_1||A_1X_1\rangle + b^*{}_1 a_1\langle A_2X_1||A_1X_1\rangle = |a_1|^2 \quad (11)$$

As sum of $P(A_1|X_1)$ and $P(A_2|X_1)$ equals to one.

$$P(A_2|X_1) = 1 - P(A_1|X_1) = |b_1|^2$$

Thus the success state leads to success and agree to play second game with probability amplitude a_1 and the success state not agree to play second game with probability amplitude b_1 respectively.

CASE-2

Consider the case of players associated with "failure in the first game-agree to play the game for second time" and "failure in the first game –do not agree to play the game for second time" can be represented as a linear combination of $|A_1X_2\rangle$ and $|A_2X_2\rangle$: So the state of mind $|\Psi_2\rangle$ associated with failure in the first game can be represented as a linear combination of $|A_1X_2\rangle$ (fail-agree to play)and $|A_2X_2\rangle$ (fail-not agree to play)is shown in the figure-3

$$|\Psi_2\rangle = a_2|A_1X_2\rangle + b_2|A_2X_2\rangle \quad (12)$$

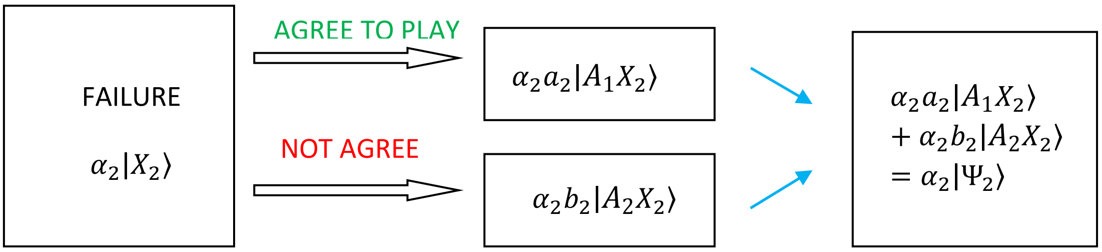

Figure-3

The normalization condition leads to $|a_2|^2 + |b_2|^2 = 1$. Again for computing fail-agree we can use the state $|\Psi_2\rangle$ and operator O_{A_1} . Hence probability of getting success in first game and agree to play second game will be:

$$P(A_1|X_2) = \langle\Psi_2|O_{A_1}|\Psi_2\rangle \quad (13)$$

$$O_{A_1}|\Psi_2\rangle = a_2 O_{A_1}|A_1X_2\rangle + b_2 O_{A_1}|A_2X_2\rangle = a_2|A_1X_2\rangle \ (\because Equn\ 6.2 - e, f)$$
$$\langle\Psi_2| = a^*{}_2\langle A_1X_2| + b^*{}_2\langle A_2X_2|$$
$$P(A_1|X_2) = \langle\Psi_2|O_{A_1}|\Psi_2\rangle = [a^*{}_2\langle A_1X_2| + b^*{}_2\langle A_2X_2|]a_2|A_1X_2\rangle$$
$$= a^*{}_2 a_2\langle A_1X_2||A_1X_2\rangle + b^*{}_2 a_2\langle A_2X_2||A_1X_2\rangle = |a_2|^2 \quad (14)$$

As sum of $P(A_1|X_2)$ and $P(A_2|X_2)$ equals to one.

$$P(A_2|X_2) = 1 - P(A_1|X_2) = |b_2|^2$$

Thus fail state leads to fail and agree to play second game with probability amplitude a_2 and the fail state not agree to play second game with probability amplitude b_2 respectively. Hence the linear combination of these states can be represented as $|\Psi_2\rangle$.

CASE-3

Let us consider the case of the probability of failure in first game and agree to play second game and the probability failure in first game and do not agree to play the game for second time: Suppose the state of mind $|\Psi\rangle$, which is representing sum of those who fails and those who are getting success in the first game i.e. the linear combination of states $|\Psi_1\rangle$ and $|\Psi_2\rangle$:

$$|\Psi\rangle = \alpha_1|\Psi_1\rangle + \alpha_2|\Psi_2\rangle \quad (15)$$

$$|\Psi\rangle = \alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle + \alpha_2 a_2|A_1X_2\rangle + \alpha_2 b_2|A_2X_2\rangle \quad (16\text{-a})$$

Where

$$d_1 = \alpha_1 a_1, d_2 = \alpha_2 a_2, d_3 = \alpha_1 b_1, d_4 = \alpha_2 b_2 \quad (16\text{-b})$$

Equation (16-a) is represented in figure-4.

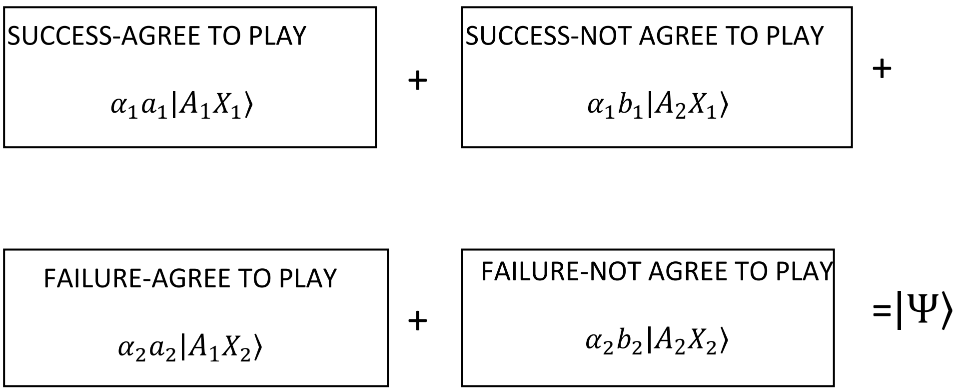

Figure-4

Equation (3) represents the states $|X_1\rangle$ and $|X_2\rangle$ in success-failure state become the state $|\Psi_1\rangle$ and $|\Psi_2\rangle$ in success-failure-agree to play-not agree to play state of equation (15) respectively. Similarly the state $|\Phi\rangle$ of equation (3) in success- failure state becomes the state $|\Psi\rangle$ in success-failure-agree to play-not agree to play state as shown in the figure-4.

V. INSPECTION OF THE PROBABILITES

Let us calculate the joint probability of getting success in first game and agree to play second game of the state $|\Psi\rangle$ as given in equation (16-a) using success operator (O_{SUC}) and agree to play (O_{A_1}) we get:

$$P(A_1X_1) = \langle\Psi|O_{A_1}O_{SUC}|\Psi\rangle$$

$$O_{SUC}|\Psi\rangle = O_{SUC}[\alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle + \alpha_2 a_2|A_1X_2\rangle + \alpha_2 b_2|A_2X_2\rangle]$$
$$(\because Equation\ (16-a))$$
$$= \alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle$$
$$(\because\ Equation\ (2-e,f,b,d)) \quad (17)$$

$$O_{A_1}O_{SUC}|\Psi\rangle = O_{A_1}[\alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle] = \alpha_1 a_1|A_1X_1\rangle = d_1|A_1X_1\rangle \quad (18)$$

$$(\because Equation\ (2-b) and\ \alpha_1 a_1 = d_1)$$
$$P(A_1X_1) = \langle\Psi|O_{A_1}O_{SUC}|\Psi\rangle = |d_1|^2 \quad (19)$$

Similarly we can find:

$$P(A_1X_2) = |d_2|^2, P(A_2X_1) = |d_3|^2, P(A_2X_2) = |d_4|^2 \quad (20)$$

Using (19) and (20)

$$P(A_1X_1) + P(A_1X_2) + P(A_2X_1) + P(A_2X_2) = |d_1|^2 + |d_2|^2 + |d_3|^2 + |d_4|^2 = 1 \quad (21)$$

Equation (19) in combination with equation (16-b) gives:

$$P(A_1X_1) = |\alpha_1|^2|a_1|^2$$

This equation with the help of equations (11) and (7) gives:

$$P(A_1X_1) = P(X_1)P(A_1|X_1) \quad (22)$$

This result was expected in equation (1-a). Similarly equations (20) can be used to obtain equations (1-b), (1-c), (1-d) respectively. Such verifications show the reliability of various probabilities computed in this section.

VI. WHEN RESULT OF FIRST GAMBLE IS NOT KNOWN

In equation (15) the state $|\Psi\rangle$ contains information regarding success and failure. In order to determine the probability of agree to play second game without any knowledge of the result of the first game, $P(A_1)$. Tversky and Shafir have taken some time between the first game and second game to clean the information of success and failure from the mind of participants. One can delete the information regarding success or failure included in the mathematical formulation by removing levels X_1 and X_2 from equation (16-a) producing a new set of equation as follows:

$$|\Psi'\rangle = d_1'|A_1\rangle + d_2'|A_1\rangle + d_3'|A_2\rangle + d_4'|A_2\rangle \quad (22(a))$$

$$|\Psi'\rangle = (d_1' + d_2')|A_1\rangle + (d_3' + d_4')|A_2\rangle \quad (22(b))$$

The constant factors d_i' of equation 22(a), where $i = 1\ to\ 4$, are same as that of d_i of equation (18). So we can write

$$|d_i'| = |d_i|, \text{where } i = 1\ to\ 4 \quad (22(c))$$

In order to find the probability of participants agreed to play the game for second time without knowing the result of success or failure in the first game, the average value of the operator O_{A_1} has to be evaluated using equation (16).

$$P(A_1) = \langle O_{A_1}\rangle = \langle\Psi|O_{A_1}|\Psi\rangle = d_1|A_1X_1\rangle + d_2|A_1X_2\rangle$$
$$= |d_1|^2 + |d_2|^2 = P(A_1X_1) + P(A_1X_2) \quad (23)$$

Similarly, if we calculate the average value of the operator O_{A_1} using equation (22-a) we will get the probability of participants agreed to play second game without knowing the result of first game as the sum of $P(A_1X_1), P(A_1X_2)$ and an interference term.

$$P(A_1) = \langle O_{A_1}\rangle = \langle\Psi'|O_{A_1}|\Psi'\rangle = |d_1' + d_2'|^2$$
$$= |d_1'|^2 + |d_2'|^2 + (d_1' * d_2') + (d_2' * d_1')$$
$$= P(A_1X_1) + P(A_1X_2) + (d_1' * d_2') + (d_2' * d_1') \quad (24)$$

Using equations (15) and (22-a) the average value of the operator $O_{A_1}$is computed to find the probability of participants agree to play the game for second time without any knowledge of success and failure. If we will compare the results obtained in equation (23) and (24) we can clearly visualize the distinction: that one finds $P(A_1)$ which is equal to sum of $P(A_1X_1)\ ,P(A_1X_2)$ and an extra interference term and in equation (23) there is no such interference terms $d_1$and d_2 i.e. we find the probability as the sum of $P(A_1X_1)\ and\ P(A_1X_2)$. This extra term was not present in the classical frame work, which is the exceptional characteristic of quantum frame work.

We can write equations (24) as:

$$P(A_1) = P(A_1X_1) + P(A_1X_2) + Q_{int}(A_1) \tag{25}$$

Where $Q_{int}(A_1) = (d_1{}' * d_2{}') + (d_2{}' * d_1{}')$ (26)

As $d_1{}'$ *and* $d_2{}'$ are complex numbers we can represent them as follows:

$$d_1{}' = |d_1{}'|exp(i\theta_1) = \left(P(A_1X_1)\right)^{\frac{1}{2}}exp(i\theta_1) \tag{27}$$

$$d_2{}' = |d_2{}'|exp(i\theta_2) = \left(P(A_1X_2)\right)^{\frac{1}{2}}exp(i\theta_2) \tag{28}$$

Where θ_1 and θ_2 are the phase angles.

Hence equation (26) will be:

$$Q_{int}(A_1) = (d_1{}' * d_2{}') + (d_2{}' * d_1{}') = 2[P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}}\cos(\theta_2 - \theta_1) \tag{29}$$

$$\left(\because we\ have\ taken\ , exp(i\theta) = \cos(\theta) + isin(\theta) and\ i = \sqrt{-1}\right)$$

As the extreme values of cosine function are ± 1 hence equation (6.29) can be written as:

$$-2[P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}} \leq Q_{int}(A_1) \leq 2[P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}} \tag{30}$$

Similarly, the operator for not agree to play second gamble is $I - O_{A_1}$, where $I =$ unit operator. Hence the probability of not agree to play second gamble without knowledge of success or failure in first game is similar to equation (25) will be:

$$P(A_2) = P(A_2X_1) + P(A_2X_2) + Q_{int}(A_2) \tag{31}$$

Where $Q_{int}(A_2) = (d_3{}' * d_4{}') + (d_4{}' * d_3{}')$ (32)

For $d_3{}' and\ d_4{}'$ we can get an equivalent equation similar to equation (29) as follows:

$$Q_{int}(A_2) = (d_3{}' * d_4{}') + (d_4{}' * d_3{}') = 2[P(A_2X_1)P(A_2X_2)]^{\frac{1}{2}}\cos(\theta_4 - \theta_3) \tag{33}$$

Combining equation (25) and (31) we get

$$P(A_1) + P(A_2) = P(A_1X_1) + P(A_1X_2) + P(A_2X_1) + P(A_2X_2) + Q_{int}(A_1) + Q_{int}(A_2) \tag{34}$$

$$\Rightarrow Q_{int}(A_1) + Q_{int}(A_2) = 0 \tag{35}$$

$$(\because Equation\ 21\ and\ 1 - e)$$

So one of the terms necessarily is positive and other one will be negative. Hence when the probability of one term will increase the probability of other term will decrease. This interference term is the pivotal feature to explain violation of sure thing principle.

VII. EXPLANATION OF TVERSKY AND SHAFIR EXPERIMENT

The experimental findings of Amos Tversky and Eldar Shafir [7] are as follows: 69% participants are agreed to play second game after knowing that they have succeeded the first game. 59% are ready for second game after knowing that they have failed first game and only 36% are ready for the second game in absence of knowing the result of first game. Hence majority of the participants prefer for second game knowing the result of first game but very few are not ready for second game without knowing the result of first game. In literature this effect is called disjunction effect of choice under uncertainty. This effect contradicts principle of the sure-thing. Now we will explain the experimental deviation from that of theory using quantum probability interference of amplitudes as shown in equation (35).

$P(X_1) =$ Probability of wining first gamble = 0.5

$P(X_2)$= Probability of not wining first gamble = 0.5

$$P(A_1|X_1) = 0.69\ , P(A_1|X_2) = 0.59$$

$$P(A_1X_1) = P(X_1)P(A_1|X_1) = 0.5 * 0.69 = 0.345$$

$$P(A_1X_2) = P(X_2)P(A_1X_2) = 0.5 * 0.59 = 0.295$$

Classically one expects:

$$P(A_1) = P(A_1X_1) + P(A_1X_2) = 0.345 + 0.295 = 0.64$$

But in experiment we get:

$$P(A_1) = 0.36$$

This anomaly can be explained using equation (25)

$$P(A_1) = P(A_1X_1) + P(A_1X_2) + Q_{int}(A_1)$$
$$\Rightarrow Q_{int}(A_1) = 0.36 - 0.64 = -0.28$$

Hence

$$Q_{int}(A_1) = -0.28$$

The factor $Q_{int}(A_1)$ is compatible with the equation (30) that suggests:

$$+0.6380 \geq Q_{int}(A_1) \geq -0.6380 \quad (36)$$

This compatibility proposes to facilitate the analysis based upon interference happening in quantum model of decision, may not be capable to predict the outcome of this gambling experiment in advance, however can clearly describe the result. The experimental findings violate sure-thing principle. On the other hand considering $\cos(\theta_2 - \theta_1)$ of equation (29) as an adjustable parameter, the data can be described by assigning

$$\cos(\theta_2 - \theta_1) = -0.439 \quad (37)$$

Hence the value of $\cos(\theta_2 - \theta_1)$ with equation (29) leads to $Q_{int}(A_1) = -0.28$ that can describe the experimental outcome for agree to play second game.

Similarly, the probability of not accepting second game, we can find the following results:

$$P(A_2|X_1) = 0.155, P(A_2|X_2) = 0.205, P(A_2) = 0.64 \quad (38)$$

$$Q_{int}(A_2) = 0.28 \; and \; \cos(\theta_2 - \theta_1) = 0.785 \quad (39)$$

Hence under uncertainty- without knowledge of success or failure in first game the decision in favour of playing the game for second time is indeed more difficult than in favour of not to play the game for second time. From equation (35), as sum $Q_{int}(A_1)$ *and* $Q_{int}(A_2)$ is zero, we can draw conclusion in advance that $Q_{int}(A_1)$ will be negative and $Q_{int}(A_2)$ will be positive. Hence we can conclude that using quantum model of decision making we are capable to describe the outcomes of two-stage gambling which was not possible in classical formalism such as principle of sure thing or classical Markov model.

VIII. DISCUSSION

- If we will put equation (29) in (24) we will get

  $$P(A_1) = P(A_1X_1) + P(A_1X_2) + [P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}} \cos(\theta_2 - \theta_1)$$

  .

  Hence in general the quantum probability can be written as

  $$P = P_1 + P_2 + 2\, cos\theta \sqrt{P_1P_2}$$

  .

  This way of representation is called quantum interference of probabilities and it is coming to the picture due to the transformation of vectors in complex Hilbert space. However the classical probability can be written as simple addition of two probabilities i.e. $= P_1 + P_2$, which shows the major difference among classical and quantum probability.

- In case of quantum decision model we can conclude, from equations (19) and (20), that the probabilities can be represented as the squares of probability amplitudes. But in case of classical decision frame work there does not exist any probability amplitude terms.
- From equation (25) we get the interference term $Q_{int}(A_1)$ naturally while calculating $|d_1' + d_2'|^2$ which is the basis to explain the paradoxical departure of principle of sure thing in case of two stage gambling experiment. This term is not found in classical decision model.
- The phase factors influence the interference term $Q_{int}(A_1)$. In classical decision making process we have not encountered the phase factors. In a limiting condition the value of the term $Q_{int}(A_1)$ found in quantum decision frame work may lead to zero. Under this

circumstances quantum result merges with the classical outcome. This is possible when the cosine term of equations (29) or (33) becomes zero. Hence classical decision model may be treated as a limiting case of quantum decision model.

IX. CONCLUSION

In this paper we have discussed the potential application of quantum interference of probability amplitudes to explain the violation of principle of sure thing in case of two stage gambling experiment conducted by Tversky and Shafir. The interference term $Q_{int}(A_1)$ is the basis to successfully explain the experiment which classical decision theory was unable to explain. A comparative discussion is made between classical and quantum model of decision making in view of the two stage gambling experiment. From our discussion we may conclude that classical decision model may be treated as a limiting case of quantum decision model.

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