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STRUCTURAL OPTIMIZATION MODEL WITH IMPRECISE RESOURCES

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ABSTRACT

The main goal of the structural optimization is to minimize the weight of structures while satisfying all design requirements imposed by design codes. In this paper, an algorithm is proposed to minimize the weight of non-linear truss structure under uncertainty. We consider a single objective nonlinear structural design model from Nicholas Ali et al.'s [14] to verify that the proposed algorithm is an effective optimization algorithm in the creation of an optimal design for truss structures. Numerical example is provided to illustrate our algorithm. The result shows that this algorithm is a powerful search and optimization technique for structural design.

KEYWORDS: *Fuzzy set theory, Non-linear programming, Fuzzy maximum decision making, Structural Optimization.*

I. INTRODUCTION

In the field of engineering [3], nonlinear structural design optimizations [1,5-9,11] are great of importance. In this regard, structural systems are described by their geometry and mechanical properties like stiffness. However, like other systems, the system description and system inputs may have uncertainties. The system uncertainty could be classified as uncertainties due to randomness or due to impreciseness. The first type of the uncertainty (randomness) is tackled by stochastic and statistical methods, the second type of the uncertainty (impreciseness) is not properly handled by this methods.

In 1965, Zadeh founded the basis of new dimension of mathematics which is now known as fuzzy set theory [4,10,13 and 18]. This theory has been used to represent uncertain or noisy information in mathematical form. Later on Bellman and Zadeh [16] used the fuzzy set theory to the decision making problem.

Nonlinear programming [15] is one of the mostly applied operations research techniques. Although it is investigated and expanded for more than five decades by many researchers from the various points of view, it is still useful to the real world problems within the framework of nonlinear programming. Fuzzy nonlinear programming (FNLP) technique [2, 17] is useful in solving problems which are difficult, impossible to solve due to imprecise information. In this paper we will discuss the concept of fuzzy decision making introduced by [12,16] and the maximum decision [19] that is used in nonlinear programming problem.

The remainder of this paper is organized in the following way. In section 2, we discuss about prerequisite fuzzy mathematics. In section 3, we discuss about fuzzy nonlinear programming**.** In section 4, we discuss about the application fuzzy nonlinear programming on two bar truss design model. Finally we draw conclusion from the results in section 4.

II. PREREQUISITE FUZZY MATHEMATICS

2.1. Fuzzy Set

Let X denote a universal set. Then the fuzzy subset A in X is a set of order pairs $A=\{x \in X : (x, \mu_A(x))\}$ where $\mu_A : X \to [0,1]$ is called the membership function which assigns a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in X$. A is non-fuzzy and $\mu_A(x)$ is identical to the characteristic function of crisp set. It is clear that the range of membership function is a subset of the non-negative real numbers

2.2. Membership function

The function $\mu(x): X \to [0,1]$ is a function with two parameters defined as

$$\mu(x) = \begin{cases} 1 & if \;\; x \le \alpha \\ \dfrac{(\alpha+\beta)-x}{\beta} & if \;\; \alpha \le x \le \alpha+\beta \\ 0 & if \;\; x \ge \alpha+\beta \end{cases}$$

It is called the trapezoidal linear membership function. This type of fuzzy number is very useful which has a large non convex fuzzy rejoin set.
Rough sketch of this type membership function is given below:

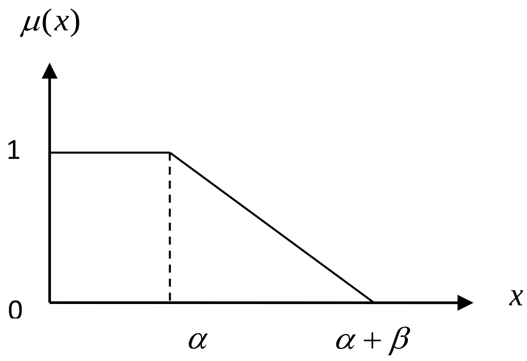

Figure 1: membership function

2.3. Fuzzy decision making

In this real world maximum decision making problems takes place in fuzzy environment. The objective goal, constraints and the consequences of possible actions are not known precisely. Under this situation Bellman et al. [16] introduced three basic concepts. They are fuzzy objective (goal), fuzzy constraints and fuzzy decision based on fuzzy objective and constraint. Now we introduced the conceptual framework for decision making in a fuzzy environment.
Let X be a given set of possible alternatives which contains the solution of decision making problem in fuzzy environment. The problem based on fuzzy decision making may be considered as follows:
Optimize a fuzzy objective function G subject to fuzzy constraints C in a space of alternatives X. The fuzzy objective G and the fuzzy constraint C is a fuzzy set on X characterized by its membership function $\mu_G : X \to [0,1]$ and $\mu_C : X \to [0,1]$ respectively. Both fuzzy objective and fuzzy constraint are desired to be satisfied simultaneously. The G and C combine to form a decision D,which is a fuzzy set resulting from intersection of G and C i.e. $D = G \cap C$. It is characterized by $\mu_D(x) = \min \{\mu_G(x), \mu_C(x)\}$

In general, if we have n goals $G_1, G_2, ..., G_n$ and m constraints $C_1, C_2, ..., C_m$, then the resultant decision can be written as follows

$$\mu_D(x) = \min\left\{\min\left\{\mu_{G_i}(x)\right\}, \min\left\{\mu_{C_j}(x)\right\}\right\}, i = 1,2,...,n;\ j = 1,2,...,m.$$

Fuzzy decision based on min-operator $D_m = \left\{x \in X : \left(x, \mu_{D_m}(x)\right)\right\}$ is a fuzzy set defined as $D_m = G \cap C$. It is characterized by $\mu_{D_m}(x) = \min\left\{\mu_G(x), \mu_C(x)\right\}$ for all $x \in X$. So maximizing decision is defined as $Max\left(\mu_{D_m}(x)\right) = \max_{x\in X}\left\{\min\left\{\mu_G(x), \mu_C(x)\right\}\right\}$.

There is another aggregation pattern, which is additive fuzzy decision. Fuzzy decision based on additive operator $D_a = \left\{x \in X : \left(x, \mu_{D_a}(x)\right)\right\}$ where $\mu_{D_a}(x) = \mu_G(x) + \mu_C(x)$ for all $x \in X$.

III. FUZZY NONLINEAR PROGRAMMING

In this section we discuss the optimization problem with nonlinear fuzzy objective and fuzzy nonlinear constraints. Consider the following nonlinear programming problem:

$$\begin{aligned} & Minimize\ f(x) \\ & subject\ to\ \ g_j(x) \le b_j\ ,\quad j = 1,2,...,m \\ & x \ge 0\ ,\quad x \in R^n \end{aligned} \qquad (3.1)$$

The fuzzy version of the problem (3.1) is

$$\begin{aligned} & Minimize\ f(x) \\ & subject\ to\ \ g_j(x)\ \le\ b_j\ ,\quad j = 1,2,...,m \\ & x \ge 0,\quad x \in R^n, \end{aligned} \qquad (3.2)$$

In problem (3.2), the tilde sign denotes a fuzzy satisfaction of the constraints. It is clear that these constraints are flexible constraints. The fuzzy Minimize corresponds to achieving the lowest possible aspiration level for the general $f(x)$. This problem can be solved by using the properties of fuzzy decision making as follows:

Step 1: Fuzzify the objective function by calculating the lower and upper bounds of the optimal values. Solve this single objective non-linear programming problem without tolerance in constraints (i.e. $g_j(x) \le b_j$), with tolerance of acceptance in constraints (i.e. $g_j(x) \le b_j + b_j^0$) by appropriate nonlinear programming technique.

Here they are

Sub-problem-1

$$\begin{aligned} & Minimize\ \ f(x) \\ & subject\ \ to \\ & g_j(x) \le b_j\ , j = 1,2,...,m; \\ & x \ge 0\ , \end{aligned} \qquad (3.3)$$

Sub-problem-2

$$\begin{aligned} & Minimize\ \ f(x) \\ & \text{subject to} \\ & g_j(x) \le b_j + b_j^0, j = 1,2,...,m; \\ & x \ge 0\ , \end{aligned} \qquad (3.4)$$

Solving (3.3) and (3.4) we may get optimal solutions $x^* = x^1,\ \ f(x^*) = f(x^1)$ and $x^* = x^2$,

$f(x^*) = f(x^2)$. Now we find lower bound (minimum) L and upper bound (maximum) U by using following rule $U = \max\left\{f(x^1), f(x^2)\right\}$. $L = \min\left\{f(x^1), f(x^2)\right\}$. Suppose M is the fuzzy set representing the objective function $f(x)$ such that $M = \left\{x \in R^n : \left(x, \mu_M(x)\right)\right\}$ where $\mu_M(x)$ is defined as:

$$\mu_M(x) = \begin{cases} 1 & if\ f(x) \le L \\ \dfrac{U - f(x)}{U - L} & if\ \ L \le f(x) \le U \\ 0 & if\ \ f(x) \ge U \end{cases}$$

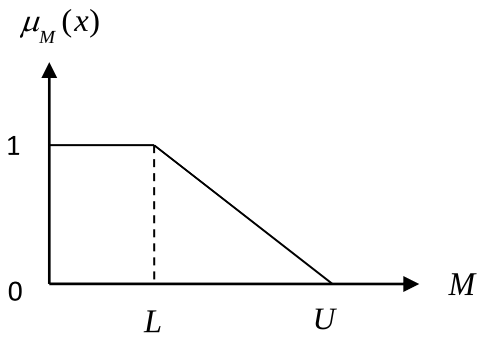

Figure 2: Rough sketch of membership function for objective

Step 2: Fuzzify the constraint $g_j(x)$, $j = 1, 2, ..., m$ .Let C_j be the fuzzy set for j^{th} constraints such that $C_j = \left\{x \in R^n : \left(x, \mu_{C_j}(x)\right)\right\}$, where $\mu_{C_j}(x)$ is defined as

$$\mu_{C_j}(x) = \begin{cases} 1 & if\ g_j(x) \le b_j \\ \dfrac{(b_j + b_j^0) - g_j(x)}{b_j^0} & if\ \ b_j \le g_j(x) \le b_j + b_j^0 \\ 0 & if\ \ g_j(x) \ge b_j + b_j^0 \end{cases}$$

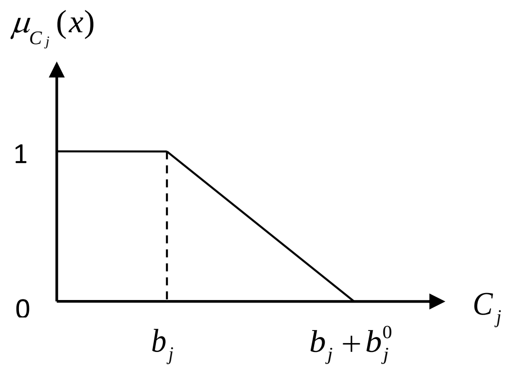

Figure 3: Rough sketch of membership function for fuzzy constraints

Step 3: Let D be the fuzzy decision set, where

$$D = M \cap C_j, \qquad j = 1, 2, ..., m.$$

(3.5)

Therefore $D = M \cap C_1 \cap C_2 \cap ..... \cap C_m$ and $D = \{x \in R^n : (x, \mu_D(x))\}$.Then we have $\mu_D(x) = \min\{\mu_M(x), \min\{\mu_{C_1}(x), ..., \mu_{C_m}(x)\}\}$.

Now we consider $\lambda = Minimize\{\mu_M(x), minimize\{\mu_{C_1}(x), ..., \mu_{C_m}(x)\}\}$

(3.6)

Then we have the optimal solution: $x^* = Maximize\ \lambda, \quad x^* \in R^n$

Step 4: Now the problem (3.2) becomes the following crisp NLP problem

$$\begin{aligned}
&Maximize\ \ \lambda \\
&subject\ \ to \\
&\quad \lambda - \mu_M(x) \le 0; \\
&\quad \lambda - \mu_{C_1}(x) \le 0; \\
&\quad .... \\
&\quad .... \\
&\quad \lambda - \mu_{C_{m-1}}(x) \le 0; \\
&\quad \lambda - \mu_{C_m}(x) \le 0; \\
&\quad \lambda \in [0,1],\ x \ge 0;
\end{aligned} \tag{3.7}$$

This is equivalent to the problem

$$\begin{aligned}
&Maximize\ \ \lambda \\
&subject\ \ to\quad \lambda - \left(\frac{U - f(x)}{U - L}\right) \le 0; \\
&\quad \lambda - \left(\frac{(b_1 + b_1^0) - g_1(x)}{b_1^0}\right) \le 0; \\
&\quad .... \\
&\quad \lambda - \left(\frac{(b_{m-1} + b_{m-1}^0) - g_{m-1}(x)}{b_1^0}\right) \le 0; \\
&\quad \lambda - \left(\frac{(b_m + b_m^0) - g_m(x)}{b_m^0}\right) \le 0; \\
&\quad \lambda \in [0,1],\ x \ge 0;
\end{aligned} \tag{3.8}$$

Solve equation (3.8) by using appropriate mathematical programming algorithm to get an optimal solution $x^* \in R^n$ and substitute in the objective function of problem (3.1). It can be easily seen that the optimal solution lies in between lower bound (minimum) L and upper bound (maximum) U i.e. $L < optimal\ value\ of\ \ f(x) < U$.

IV. Application On Two Bar Truss

A two-bar truss shown in Figure 4 is designed to support the loading condition. The structure is subject to constraints in geometry, area, stress [14].

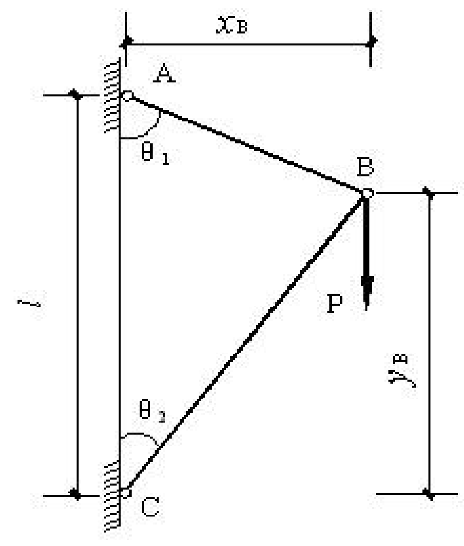

Figure 4: Design of the two-bar planar truss

The Optimization model of the two-bar truss is as follows:

$$
\begin{aligned}
&\textit{Minimize } WT\left(A_1, A_2, y_B\right) = \rho\left(A_1\sqrt{x_B^2+(l-y_B)^2} + A_2\sqrt{x_B^2+y_B^2}\right) \\
&\textit{subject to.}\quad g_1(A_1,A_2,y_B) \equiv \frac{P\sqrt{x_B^2+(l-y_B)^2}}{lA_1} \le [\sigma_t]; \\
&\qquad g_2(A_1,A_2,y_B) \equiv \frac{P\sqrt{x_B^2+y_B^2}}{lA_2} \le [\sigma_c]; \\
&\qquad 0.5 \le y_B \le 1.5 \qquad A_1 > 0; A_2 > 0;
\end{aligned}
\tag{4.1}
$$

where weight $=WT$,Nodal load = P , Volume density = ρ ,Length $AC = l$, ;Perpendicular distance from AC to point B= x_B Allowable tensile stress =$[\sigma_t]$ .Allowable compressive stress = $[\sigma_c]$,Cross sectional area of AB bar = A_1 ,Cross-sectional area of BC bar $= A_2$, y coordinate of node B = y_B .

The input data for the structural optimization problem (4.1) is given as follows: Nodal load (P) =100KN; Volume density $(\rho) = 7.7KN/m^3$; Length (l) =2000 mm ;Width (x_B) =1000 mm ; Allowable tensile stress $[\sigma_t]$ =130 MPa; Allowable compressive stress $[\sigma_c]$ = 90 MPa ; y coordinate of node B $(y_B) = (500\,mm \le y_B \le 1500\,mm)$ .The nonlinear structural optimization problem of the two-bar truss is

$$
\begin{aligned}
&\textit{Minimize } WT(A_1,A_2,y_B) = 7.7\left(A_1\sqrt{1+(2-y_B)^2} + A_2\sqrt{1+y_B^2}\right) \\
&\textit{subject to } g_1(A_1,A_2,y_B) \equiv \frac{100\sqrt{1+(2-y_B)^2}}{2A_1} \le 130 \text{ ;} \\
&\qquad g_2(A_1,A_2,y_B) \equiv \frac{100\sqrt{1+y_B{}^2}}{2A_2} \le 90 \text{ ;} \\
&\qquad 0.5 \le y_B \le 1.5 \text{ ; } \quad A_1 > 0; A_2 > 0;
\end{aligned}
\tag{4.2}
$$

The optimal solution is $A_1^* = 0.5954331 m^2, A_2^* = 0.7178118 m^2, y_B^* = 0.8181818 m$. Therefore $WT^* = 14.23932 KN$ satisfies the constraints of problem (4.2). Now the fuzzy version of the problem is

$$\begin{aligned} &Minimize\ WT(A_1, A_2, y_B) = 7.7\left(A_1\sqrt{1+(2-y_B)^2} + A_2\sqrt{1+y_B^2}\right) \\ &subject\ to\ g_1(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+(2-y_B)^2}}{2A_1} \le 130\ with\ tolerance\ 20; \\ &g_2(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+y_B^2}}{2A_2} \le 90\ with\ tolerance\ 10; \\ &0.5 \le y_B \le 1.5;\quad A_1 > 0; A_2 > 0; \end{aligned} \tag{4.3}$$

Therefore, $b_1 = 130$, $b_2 = 90$, $b_1^0 = 20$ and $b_2^0 = 10$. Now we can find lower bound L and upper bound U by solving the two crisp NLPP as follows:

Sub-Problem-1:

$$\begin{aligned} &Minimize WT(A_1, A_2, y_B) = 7.7\left(A_1\sqrt{1+(2-y_B)^2} + A_2\sqrt{1+y_B^2}\right) \\ &subject\ to\ g_1(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+(2-y_B)^2}}{2A_1} \le 130; \\ &g_2(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+y_B^2}}{2A_2} \le 90; \\ &0.5 \le y_B \le 1.5;\quad A_1 > 0; A_2 > 0; \end{aligned} \tag{4.4}$$

Since the problem is the same first problem (4.2) and they have the same solution $WT_1 = 14.23932 KN$.

Sub-Problem-2:

$$\begin{aligned} &Minimize\ WT(A_1, A_2, y_B) = 7.7\left(A_1\sqrt{1+(2-y_B)^2} + A_2\sqrt{1+y_B^2}\right) \\ &subject\ to\ g_1(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+(2-y_B)^2}}{2A_1} \le 150; \\ &g_2(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+y_B^2}}{2A_2} \le 100; \\ &0.5 \le y_B \le 1.5;\quad A_1 > 0; A_2 > 0; \end{aligned} \tag{4.5}$$

The solution is $A_1^* = 0.5206833 m^2, A_2^* = 0.6403124 m^2, y_B^* = 0.80 m$. Finally, $WT_2 = 12.57667 KN$. Therefore, $U = \max\{WT_1, WT_2\}$ and $L = \min\{WT_1, WT_2\}$.

Let M be the set of objective function such that

$$M = \left\{A_1, A_2, y_B \in R^n : \left(x, \mu_M\left(A_1, A_2, y_B\right)\right)\right\}$$

and

$$\mu_M(A_1, A_2, y_B) = \begin{cases} 1 & if\ WT(A_1, A_2, y_B) \le 12.57667 \\ \dfrac{14.23932 - WT(A_1, A_2, y_B)}{1.66265} & if\ 12.57667 \le WT(A_1, A_2, y_B) \le 14.23932 \\ 0 & if\ WT(A_1, A_2, y_B) \ge 14.23932 \end{cases}$$

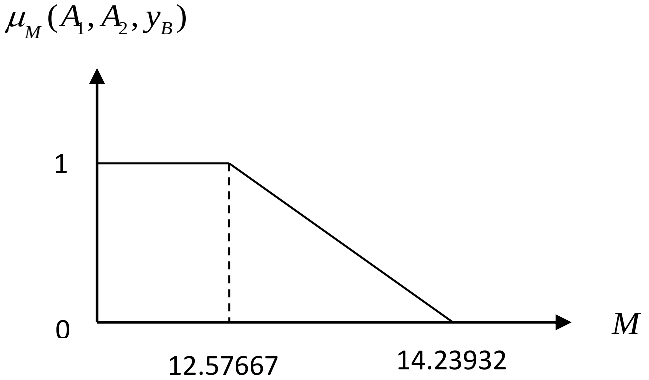

Figure 5: Rough sketch of membership function for fuzzy objective

In addition, let C_1 be the set for first constraint for $g_1(A_1, A_2, y_B)$ such that

$$C_1 = \left\{ A_1, A_2, y_B \in R^n : \left(x, \mu_{C_1}\left(A_1, A_2, y_B \right) \right) \right\}$$ where

$$\mu_{C_1}(A_1, A_2, y_B) = \begin{cases} 1 & if\ g_1(A_1, A_2, y_B) \le 130 \\ \dfrac{150 - g_1(A_1, A_2, y_B)}{20} & if\ 130 \le g_1(A_1, A_2, y_B) \le 150 \\ 0 & if\ g_1(A_1, A_2, y_B) \ge 150 \end{cases}$$

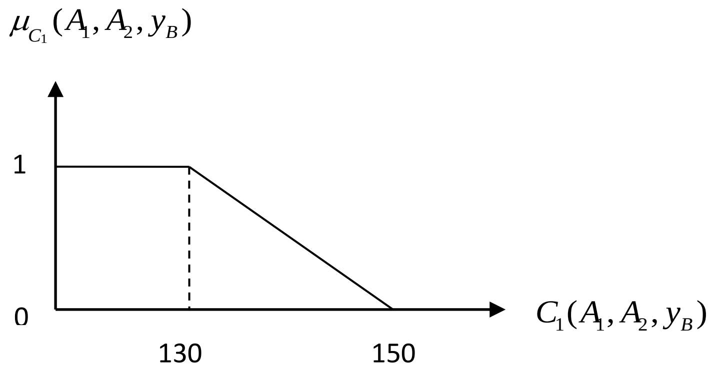

Figure 6: Rough sketch of membership function for fuzzy constraints $C_1(A_1, A_2, y_B)$

and C_2 be the fuzzy set for second constraint $g_2(A_1, A_2, y_B)$ such that

$$C_2 = \left\{ A_1, A_2, y_B \in R^n : \left(x, \mu_{C_2}\left(A_1, A_2, y_B \right) \right) \right\}$$ where

$$\mu_{C_2}(A_1,A_2,y_B)=\begin{cases}1 & if\ \ g_2(A_1,A_2,y_B)\le 90\\ \dfrac{100-g_2(A_1,A_2,y_B)}{10} & if\ 90\le g_2(A_1,A_2,y_B)\le 100\\ 0 & if\ \ g_2(A_1,A_2,y_B)\ge 100\end{cases}$$

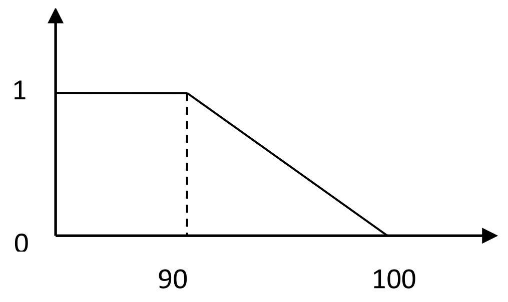

Figure 7: Rough sketch of membership function for fuzzy constraints $C_2(A_1,A_2,y_B)$

The fuzzy decision making for this problem is

$$\mu_D(A_1,A_2,y_B)=\min\left\{\mu_M(A_1,A_2,y_B),\min\left\{\mu_{C_1}(A_1,A_2,y_B),\mu_{C_2}(A_1,A_2,y_B)\right\}\right\}$$

For $\lambda=\min\left\{\mu_M(A_1,A_2,y_B),\min\left\{\mu_{C_1}(A_1,A_2,y_B),\mu_{C_2}(A_1,A_2,y_B)\right\}\right\}$, with optimal decision $A_1^*,A_2^*,y_B^*=Maximize\ \lambda$.

Finally, the crisp NLP corresponding with the fuzzy NLP is given by

$$\begin{aligned}&Maximize\ \ \lambda\\ &subject\ \ to\\ &\quad\lambda-\mu_M(A_1,A_2,y_B)\le 0;\\ &\quad\lambda-\mu_{C_1}(A_1,A_2,y_B)\le 0;\\ &\quad\lambda-\mu_{C_2}(A_1,A_2,y_B)\le 0;\\ &\quad\lambda\in[0,1],\ A_1,A_2,y_B>0;\end{aligned}$$

Which is equivalent to the following problem is

$$\begin{aligned}&Maximize\ \ \lambda\\ &subject\ \ to\\ &\quad\lambda-\left(\frac{14.23932-WT\left(A_1,A_2,y_B\right)}{1.66265}\right)\le 0;\\ &\quad\lambda-\left(\frac{150-g_1(A_1,A_2,y_B)}{20}\right)\le 0;\\ &\quad\lambda-\left(\frac{100-g_2(A_1,A_2,y_B)}{10}\right)\le 0;\\ &\quad\lambda\in[0,1],\ A_1,A_2,y_B>0;\end{aligned}\tag{4.6}$$

Therefore, the solution of problem (4.6) is $A_1^* = 0.5567145\,m^2, A_2^* = 0.6780355\,m^2, y_B^* = 0.8087956\,m$. Now we can submit A_1^*, A_2^* *and* y_B^* in the objective function of the crisp NLP. It can be obtained $WT_{optimal} = 13.38188\ KN$, where $L < WT_{optimal} < U$. Clearly, in comparison the crisp problem we have more accurate solution.

V. CONCLUSION

The described method, as illustrated, is efficient and reliable. Furthermore, it is proposed that the results solution of fuzzy optimization is a generalization of the solution of the crisp optimization problem. The numerical result shows that the solutions in fuzzified problems are more accurate then results in crisp problems. Here decision maker may obtain the optimal results according to his/her expectation. The method presented is quite general and can be applied to the model in other areas of operations research and other engineering field of optimization involvement.

Conflict of interest: The authors declare that there is no conflict of interest.

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