

# LIMITATION ON PARTITIONING OF PROBABILITY SPACE IN ENTROPY BASED DETECTION FOR SPECTRUM SENSING IN COGNITIVE RADIO NETWORKS

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## ABSTRACT

*The limited available spectrum and the inefficiency in the spectrum usage lead to the requirement for a new communication paradigm to exploit the existing wireless spectrum opportunistically. This new paradigm is the Cognitive Radio Networks (CRN). However, the existing spectrum sensing techniques being envisaged for detecting the presence of primary users in CRN has a fundamental limitation and that is, sensitivity to noise uncertainty. Noise uncertainty rapidly deteriorates the performance of traditional detectors such as matched filters, energy detectors and even cyclostationary detectors. Only Entropy Based Detection (EBD) is robust against noise uncertainty. In EBD, the noise uncertainty is transferred to the fluctuations in the bin width by fixing the partitioning of the probability space as bin number and is represented by  $L$ . In this paper it is mathematically explained how this transfer takes place. In this paper it has been proved that though bin number is the design parameter of an EBD system, it is bounded by limitation in terms of higher limits and lower limits. To prove the concept, simulation results of Monte Carlo experiments for spectrum sensing have been carried out on MatLab software and the same have been illustrated in this paper.*

**KEYWORDS:** Cognitive radio networks (CRN), Entropy based detection (EBD) spectrum sensing, noise uncertainty, probability of detection ( $P_D$ ), probability of false alarm ( $P_F$ ), dimension of the probability space ( $L$ ).

## I. INTRODUCTION

CRN has been a major research topic [1] since its introduction by Mitola in 2000s [2][3]. Sensitivity to noise uncertainty is a fundamental limitation of the existing spectrum sensing techniques in cognitive radio networks (CRN). Because of noise uncertainty, the performance of even supposedly efficient traditional detectors such as energy detectors and matched filter detectors deteriorate rapidly at low Signal-to-Noise Ratios (SNR). SNR levels for these detectors are as follows: matched filter detector (-13.8 dB) and energy (-12.4 dB) [4]. Without accurate estimation of background noise, an absolute “SNR wall” exists below which a detector may fail to be robust, no matter how long the detector can observe the channel [5]. Noise uncertainty can be alleviated by on-line calibration, but it cannot be completely eliminated. The SNR wall problem might be overcome by macro scale features assuming that the prior knowledge of channel characteristics and infinite sample size exists [6]. However, these assumptions usually do not hold in practice. To counteract noise uncertainty, in [7] an entropy-based spectrum sensing scheme is introduced. In [4] the entropy of the sensed signal is estimated in the frequency domain with a probability space partitioned into fixed dimensions ( $L$ ) where  $L$  proves to be an important design parameter while spectrum sensing in CRN. This scheme is robust against noise uncertainty. Simulation results confirm the robustness of this scheme and show 6dB and 5dB performance improvement compared with energy detectors and cyclostationary detectors, respectively. Hence, in a scenario where the signal received is weak due to multipath fading or long distances or high region of electromagnetic interference; an entropy based detection scheme is more efficient, as it detects a primary user even in low SNR conditions.

In this paper Monte Carlo experiments on spectrum sensing for CRN using Entropy based detection

are carried out with respect to  $P_D$ ,  $P_F$  [8], the spectrum sensing time and the dimension of the probability space. It has been shown that the upper and lower bound of probability space partition plays an important role in satisfactory detection performance. Based on ROC, the performance of Entropy based detection for BPSK and QPSK modulation formats are also analyzed.

Section 2 describes EBD spectrum sensing methodology used for obtaining the results. In this section, it has been explained using mathematical expressions as to how EBD system proves to be robust to noise uncertainty.

Section 3 illustrates the results and its analysis. The results have been obtained by carrying out Monte Carlo experiments for spectrum sensing on MatLab software.

Section 4 winds up this paper with the conclusions which brings out the outcomes of the simulation results and the concept hence proved.

## II. EBD SPECTRUM SENSING METHODOLOGY

Figure 1 shows the EBD spectrum sensing technique for CRN. The received signal  $y(n)$  is sampled followed by DFT transformation. Then its entropy is calculated and the value is compared with a threshold value. The result of comparison decides the presence or the absence of the primary user.  $H_L(R)$  is the calculated entropy and  $\lambda$  is the threshold for comparison.

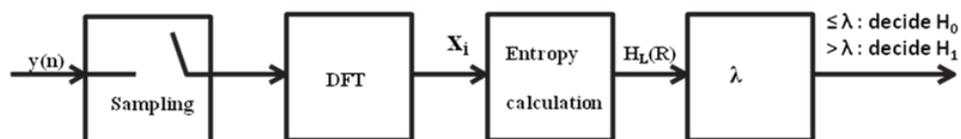


Figure 1. EBD spectrum Sensing for CRN

In case of EBD Spectrum Sensing, for eliminating the noise uncertainty factor while carrying out mathematical analysis, entropy of the received signal is calculated in frequency domain. For analysis, the discrete signal  $y(n)$  is taken with frequency  $f_c$  and bandwidth  $B_w$ . It is assumed that the signal is received by the cognitive receiver after sampling and can be expressed as:

$$y(n) = x(n) + w(n) \text{ where } n=0,1,2,3 \dots N-1 \quad (1)$$

where  $x(n)$  is the primary signal which the cognitive receiver wants to detect and  $w(n)$  is the White Gaussian Noise (WGN). Assume the variance of this WGN to be  $\sigma_0^2$ . The primary signal  $x(n)$  can be assumed as any stochastic signal which exhibits and accounts for channel characteristics like multipath fading.

The spectrum sensing methodology is formulated as binary hypothesis  $H_0$  and  $H_1$ .  $H_0$  denotes that the primary user is not present and hence the frequency spectrum which is left unutilized by it can be exploited by cognitive radio users.  $H_1$  denotes that the primary user is present and hence the spectrum being used by the cognitive users has to be vacated immediately so that it does not cause interference to the primary user network.

For frequency domain analysis, we apply Discrete Fourier Transform (DFT) to the signal given in equation (1) and obtain a complex spectrum of the received signal for a  $N$  point DFT.  $H_0$ , indicates absence of primary user and consists of only WGN with a zero mean and variance expressed as  $\sigma_0^2/N$ . This variance is the factor which is responsible for noise uncertainty. Let  $R$  denote the random variable which represents the spectrum magnitude of the measured signal. Estimation of the probability density function (PDF) for  $R$  is required. It follows Rayleigh distribution with parameter  $\sigma_1 = \sigma_0/\sqrt{2N}$  and differential entropy  $H_d$  given as :-

$$H_d(R) = 1 + \ln \frac{\sigma_1}{\sqrt{2}} + \gamma/2 \quad (2)$$

where  $\gamma$  is Euler-Mascheroni constant.

Spectrum magnitude of the received signal in  $H_1$  follows Rice distribution without analytical

expression of differential entropy. For a given dimension of the probability space  $L$ , the bin width  $\Delta_b$  is also a random variable with  $E(\Delta_b) = \sigma_1/L$ . Now the entropy of the quantized version can be written as

$$\begin{aligned}
 H_L(R) &= - \sum_{i=1}^L p_i \log p_i \\
 &= - \sum_{i=1}^L (f(R_i)\Delta_b) \log(f(R_i)\Delta_b) \\
 &= - \sum_{i=1}^L (f(R_i)\Delta_b) \log(f(R_i)) - E(\log \Delta_b)
 \end{aligned} \tag{3}$$

With natural logarithm, we obtain

$$H_L(R) \cong H_d(R) - \ln(E(\Delta_b))$$

If the density  $f(R)$  of the random variable  $R$  is Riemann integrable, the first term in equation (3) can be approximated by the differential entropy [9].

Substituting the value of differential entropy from equation (2),  $H_L(R)$  is given by,

$$\begin{aligned}
 H_L(R) &\cong 1 + \ln \frac{\sigma_1}{\sqrt{2}} + \gamma/2 - \ln \frac{\sigma_1}{L} \\
 &= \ln \frac{L}{\sqrt{2}} + \frac{\gamma}{2} + 1
 \end{aligned} \tag{4}$$

In equation (4), it is seen that the noise power term  $\sigma_1^2$  is cancelled out and discrete entropy is approximated by a constant for a given bin number  $L$ , which implies that the false alarm ratio is almost fixed for a given threshold. It is observed that the quantized entropy is independent of the variance factor which is mathematically eliminated as seen in the above equations. Hence, it is shown that entropy based detection is robust to noise uncertainty.

Noise uncertainty is thus transferred to fluctuations of the bin width  $\Delta_b$  by fixing the bin number  $L$ . Therefore, the bin number  $L$  becomes a design parameter determined by nominal noise, while the entropy does not depend on the noise term as shown in equation (4).

Because of quantization errors and the ideality of the Gaussian noise model, the noise term cannot be perfectly cancelled out in practice. Assuming that the estimated noise entropy follows a Gaussian distribution with theoretical value  $H_L$  in equation (4) as the mean value and variance  $\sigma_e^2$  (for calculated entropy), the threshold is determined by

$$\lambda = H_L + Q^{-1}(1 - P_F)\sigma_e \tag{5}$$

where  $\lambda$  is the detection threshold determined by a target false alarm ratio  $P_F$  and where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp(-\tau^2/2) d\tau$$

Different bin numbers reveal different features of the data and  $L > 10$  is required in the detection for a better description of the received signal [6]. Once the bin number is fixed, the entropy in  $H_0$  is a constant independent of sample size and the false alarm ratio being invariant. However, the sample size affects detection probability in  $H_1$ . For a given sensing time period, detection performance can be improved by increasing the sampling frequency.

In section III, the results of Monte Carlo experiments on spectrum sensing for CRN using entropy based detection are simulated to determine PD, PF and the dimension of the probability space. The bound of probability space partition for satisfactory detection performance is analyzed. ROC performance of Entropy based detection for BPSK and QPSK modulation formats are also analyzed.

### III. RESULT AND ANALYSIS FOR EBD

To substantiate the concept as brought out in section II, Monte Carlo experiments are carried out with each result averaged over 10000 runs. Assumed parameters are  $L=15$ ,  $P_F = 0.08$  and  $\sigma_e^2 / H_L = 10^{-3}$ . For these parameters, the theoretical entropy of noise is  $H_L=2.198$  as obtained from equation (4) and test threshold  $\lambda = 2.149$  obtained from equation (5).

The value of  $\lambda$  can be varied by varying the dimension of the probability space. The primary signal is a binary phase shift keying (BPSK) modulated signal with symbol ratio  $R_s$ , carrier frequency  $f_c$ , sampling frequency  $f_s$  and sampling time  $t$  in msec. These parameters are varied to obtain various simulation results.

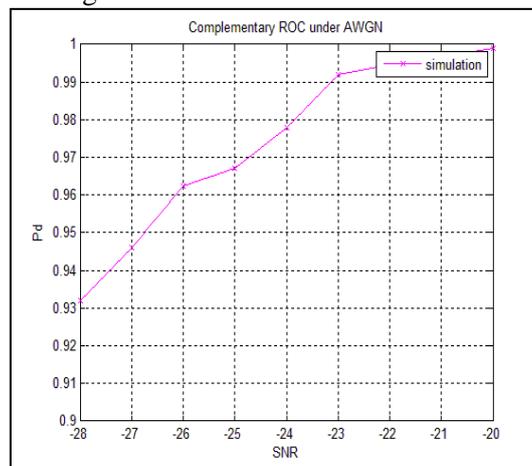
Table 1, shows the  $P_D$  and  $P_F$  values for different dimensions of the probability space ( $L$ ) with resulting sensing time  $T$  normalized to msec.

**Table 1:**  $P_D$  and  $P_F$  For Various Probability Space Dimensions and Sensing Duration

Probability Space Dimension (L)	Sensing Time, T (msec)	$P_F$	$P_D$
10	6	0.1	0.88
15	8	0.07	0.83
20	10.6	0.05	0.79
25	13.1	0.04	0.76
30	16	0.033	0.72
35	18	0.0285	0.72
40	20.3	0.025	0.70
45	22.6	0.022	0.69
50	25	0.02	0.64

It is observed from Table 1 that as the value of  $L$  increases the value of  $P_D$  and  $P_F$  decreases. Decrease in the value of  $P_F$  is desirable but the value of  $P_D$  should not decrease. Value of  $P_D$  cannot fall below a certain threshold. This becomes the limiting factor for the higher value of  $L$ , which has been proved as an important design parameter for EBD in spectrum sensing in CRN.

From Table 1, it is also observed that as we increase the value of  $L$ , the spectrum sensing time also increases. This cannot exceed a certain threshold value. A minimum sensing time has to be fixed within which the receiver must detect the primary user and vacate the spectrum, otherwise it will result in causing interference to the primary radio network. The sensing time may vary for different computing hardware and simulating software.



**Figure 2.** ROC for entropy based detection with BPSK modulation

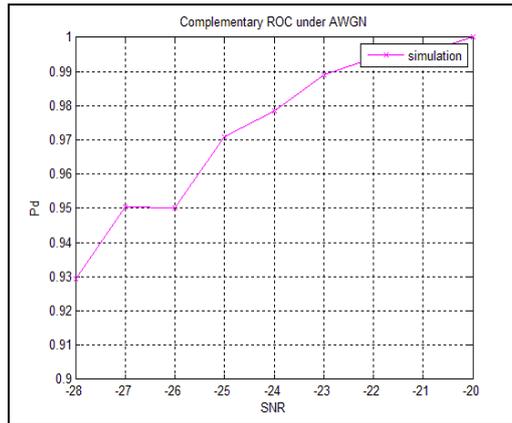


Figure 3. ROC for entropy based detection with QPSK modulation

From Figure 2 and Figure 3 as shown above, it is observed that ROC gives a comparatively better curve for BPSK than for QPSK modulation. For BPSK modulation, the PD is slightly higher. Hence, it can be deduced that EBD performs better with BPSK modulation format.

Figure 4 and Figure 5 shows the simulation results for the ROC performance of the EBD technique for spectrum sensing for  $N=2000$  and  $N=5000$ . The input signal to the EBD is an average value over  $N$ . Here,  $N$  is the total number of signals taken to average the input signal given to the CR receiver. Higher values gives better average and hence better results.

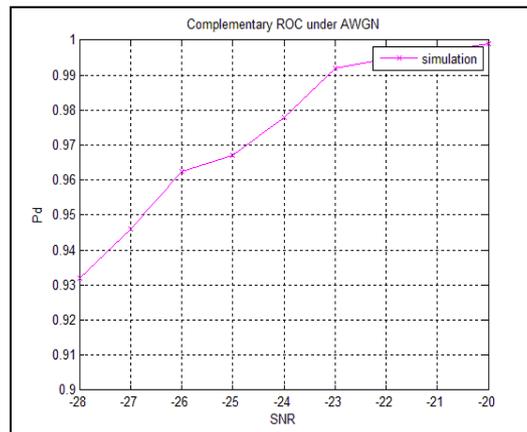


Figure 4. ROC for entropy based detection with BPSK modulation with  $N=2000$

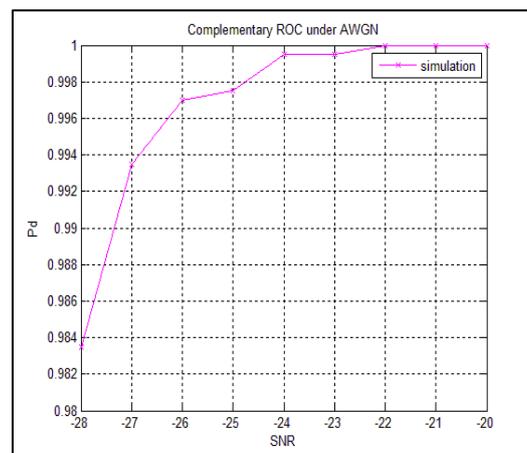


Figure 5. ROC for entropy based detection with BPSK modulation with  $N=5000$

From Figure 4 and Figure 5, it is observed that for higher averages, that is, for higher value of  $N$ , the

ROC curve is better for BPSK modulation and the PD value improves significantly. With EBD, it is possible to detect signal even with low SNR [6]. Further, in this paper, it has been established that there is a trade-off between the value of L and the detection performance of EBD technique. The probability space for detection cannot be partitioned beyond a certain threshold. It has also been observed from the ROC graphs that EBD will perform better for BPSK modulation format. Finally, higher are the numbers of signal being averaged before providing an input to the CR receiver, better are the obtained results.

#### **IV. CONCLUSION**

Numerous advances have been made in the field of CRN [10]. Most of the research carried out in this technology is focused on efficient spectrum sensing technique. In this paper, simulation results using Monte Carlo experiments on spectrum sensing for CRN using EBD are given for PD, PF, the spectrum sensing time and the dimension of the probability space (L). Also the performance of EBD for BPSK and QPSK modulation formats are analyzed based on ROC.

From the result analysis, it has been established that there is a trade-off between the value of L and the detection performance of EBD technique. The probability space for detection cannot be partitioned beyond a certain threshold. As shown in Table 1, the detection performance deteriorates with increase in the number of partition of the probability space. Say, if the threshold decided by the CR user is  $PD=0.69$ , then for  $L>45$ , detection will fail. Hence the probability space will not be partitioned into more than 45 bins.

From Figures 2 and 3, it is evident that EBD will perform better for BPSK modulation format rather than QPSK. As seen from the ROC,  $PD=0.933$  for BPSK modulation and  $PD=0.93$  for QPSK modulation which shows that the detection performance of EBD is better for BPSK.

Finally, Figure 4 and 5 shows that higher are the numbers of signal being averaged before providing an input to the CR receiver, better are the obtained results. For EBD with BPSK modulation, for  $N=2000$ , the value of  $PD=0.933$  whereas for  $N=5000$ , the value of  $PD=0.983$ . Thus, it is established that EBD will perform better for higher value of N.

Hence, in this paper, it has been established that the value of L can be increased till the time the probability of detection is higher than the threshold. The value of L is also governed by the sensing time fixed for spectrum sensing. In CRN, EBD will perform better for BPSK modulation format rather than for QPSK. Finally it was shown that when more number of signal samples (N) are taken, the average input given to the CR receiver will provide better results.

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