BLOOD FLOW RESISTANCE FOR A SMALL ARTERY WITH THE EFFECT OF MULTIPLE STENOSES AND POST STENOTIC DILATATION

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ABSTRACT

An attention for the effect of a post stenotic dilatation and multiple stenoses on an arterial blood flow is made through this work. The resistance to flow ratio is investigated with the different values of yield stress, viscosity, and the flux. For this study we consider fully developed, one-dimensional non-Newtonian fluid namely Bingham plastic fluid flow through a single vessel. The radius of vessel is varying. The formula for the resistance to flow is determined and the results are found for the yield stress, viscosity and flux. The numerical values of the resistance to flow ratio vary from maximum radii of the two abnormal segments to the minimum radii.

KEYWORDS: *Bingham plastic fluid, stenosis, dilatation, yield stress, viscosity, resistance flow, flux*.

I. INTRODUCTION

Stenosis or arteriosclerosis, which means narrowing of anybody passage (tube), is thus an abnormal and unnatural increase in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions. Although the genesis of such diseases remains unknown, there is a strong belief that hydrodynamic factors play a very significant role in the formation and proliferation of the disease. The deposition of cholesterol and proliferation of the connective tissues in the arterial wall form plaques which grow inward and restrict blood flow.

One of the phenomenon of stenosis are common in the circulatory system but particular importance in the coronary due to the development of the artery, and the development of arterioscleritic plaque or other type of abnormal tissue development. The development of stenosis in an artery can have serious consequence and it can be disturbed the normal functioning of the circularity system. In particular, it may lead to increase resistance to flow, tissue damage leading to post stenosis dilatation and increase danger of complete obstruction. Stenoses in the arteries of mammals are common occurrence, and for many years researchers have endeavored to model the flow of blood through stenosed arteries experimentally and theoretically. It is known that arterial diseases are main cause of death in most of the western world.

As far as the research in this direction is concerned, Young [1] investigated the effects of a time-dependent stenosis on flow through a tube while Imacda *et al.* [2] made an analysis for the non linear pulsatile blood flow in arteries, and Mishra *et al.* [3] investigated the flow in arteries in the presence of stenosis. Mann *et al.* [4] discussed about the flow of non-Newtonian blood analog fluids in rigid curved and straight artery models. Again Tandon *et. al.* [5] described a model for blood flow through stenotic tube, while Majhi and Nair [6] discussed about Pulsatile flow of third grade fluids under body acceleration modelling blood flow. Dash *et al.* [7] also studied the effect of body acceleration on the flow field in a porous stenotic artery.

Again Chakravarty and Sannigrahi [8] reported a study of an analytical estimate of the flow field in a porous stenotic artery subject to body acceleration where it was observed that when the body acceleration is withdrawn from the system under study the wall shear stresses are enhanced largely throughout the constricted arterial segment under consideration. Berger and Jou [9] have shown that the relationship between flow in arteries and the sites where atherosclerosis develops are linked. It is

well known that the flow of blood depends on the pumping action of the heart which gives blood flow its pulsatile nature. Although Secomb *et al.* [10] discussed the blood flow and red blood cell deformation in non uniform capillaries effects of the endothelial surface layer, while Anand, *et al.* [11] worked on a shear thinning visco-elastic fluid model for describing the flow of blood. Kumar *et al.* [12] worked on the numerically study of the axi-symmetric blood flow in a constricted rigid tube, while Bali, *et al.* [13] observed the effect of a magnetic field on the resistance of blood flow through stenotic artery. Hayat *et al.* [14] studied about the influence of slip on the peristaltic motion of a third order fluid in an asymmetric channel, while again Kumar *et al.* [15] reported the results of oscillatory MHD flow of blood through an artery with mild stenosis. Assuming blood to be an incompressible biviscous fluid, the effect of uniform transverse magnetic field on its pulsatile motion through an axisymmetric tube was analyzed by Sanyal and Biswas [16]. The influence of slip condition on peristaltic transport of a compressible Maxwell fluid through porous medium in a tube has been studied by Eldesoky [17]. Recently Kumar and Diwakar [18] worked on a biomagentic fluid dynamic model for the MHD couette flow between two infinite horizontal parallel porous plates and observed that the main flow component decreases with the increase of hartmann number and the velocity decreases with the increases of the injection suction parameter, they also investigate the effects of hartmann number and suction parameter along with the velocity and temperature distribution. In same direction Kumar and Diwakar [19] also discussed about a mathematical model for newtonian blood flow in the presence of applied magnetic field and found that results concerning the velocity and temperature field, and rate of heat transfer indicate that the presence of magnetic field appreciable influence the flow field, while the flow is appreciably influenced by the application of the magnetic field and in particularly by the strength and the magnetic field gradient.
The studies mentioned above are a few out of the literature on physiological fluid dynamics. The studies mentioned above all treat blood as a Newtonian fluid but as suggested in blood behaves like a non-Newtonian fluid under certain conditions. Therefore in this study we propose a mathematical model for pulsatile blood flow treating blood as a non-Newtonian power law fluid. Our main motto is to find out the resistance ratio for fully developed Bingham plastic flow through a multiple stenoses and post dilatation artery.

II. MATHEMATICAL MODELING

We consider an axially symmetric, laminar, incompressible and fully developed flow of blood in the z direction through a circular artery which containing multiple abnormal segments in which the blood flows, stenosis as shown in Fig 1. We have used cylindrical polar coordinates whose origin is located on the axis of stenosed artery. The wall of the stenosed artery is assumed to be rigid It can be shown that the radial velocity is negligibly small and can be neglected. If w is the axial velocity, τ is the shear stress, τ_0 is the yield stress and μ is the viscosity of the blood then in this case, the constitutive equation for the fluid are:

$$e = f(\tau) = -\frac{dw}{dr} = \left\{ \begin{array}{ll} \dfrac{\tau - \tau_0}{\mu} & for\ \ \tau \geq \tau_0 \\ 0 & for\ \ \tau \leq \tau_0 \end{array} \right\} \qquad \ldots(i)$$

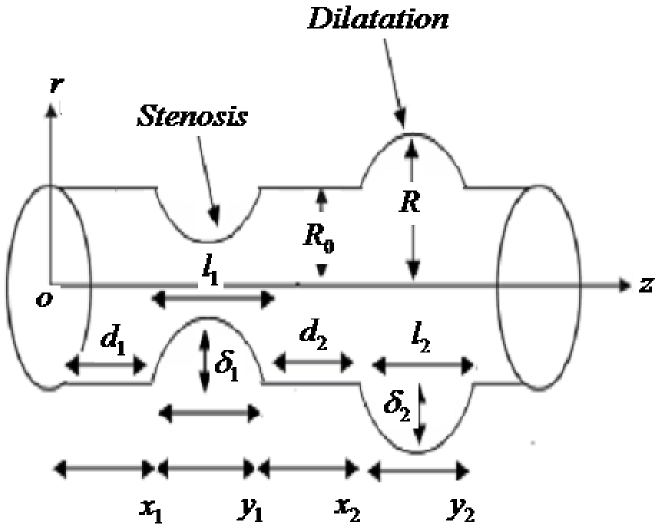

Fig 1 Schematic diagram of artial segment

The geometrical description of the figure by the equation

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{\delta_i}{R_0}\left\{1 + \cos\frac{2\pi}{l_i}\left(z - x_i - \frac{l_i}{z}\right)\right\} & for\ x_i \le z \le y_i \\ 1 & otherwise \end{cases} \quad \ldots(ii)$$

Where δ_i is the maximum distance at i^{th} abnormal segment projects into the lumen and negative for the aneurysms and positive for stenosis. R_0 is the radius of normal artery, R is the radius of artery at dilatation. l_i is the length of the i^{th} abnormal segment x_i is the radius from the origin to the start of the i^{th} abnormal segment and y_i is the distance from the origin to the end of the i^{th} abnormal segment and they are given by:

$$x_i = \left(\sum_{j=1}^{i}(d_j + l_j) - l_0\right) \quad \ldots(iii)$$

and

$$y_i = \left(\sum_{j=1}^{i}(d_j + l_0)\right) \quad \ldots(iv)$$

III. MATHEMATICAL FORMULATION AND SOLUTION

If we consider the velocity parallel to the axis and it is the function of r only and expected to velocity is maximum on the axis and zero on the surface so that only non zero component of the strain rate is

$$e = -\left(\frac{dw}{dr}\right) \quad \ldots(v)$$

for the non-Newtonian fluid

$$\tau = f(e)$$

$$\tau = f\left(-\frac{dw}{dr}\right) \quad \ldots(vi)$$

and the expression for τ is

$$\tau = \frac{1}{2}\Pr \quad \text{...(vii)}$$

From equation (vi) and (vii) we have

$$-\frac{1}{2}\Pr = f\left(-\frac{dw}{dr}\right) \quad \text{...(viii)}$$

Now the flux Q through the artery can be written in the following form

$$Q = \int_0^R 2\pi r w dr$$

on integrating and applying no slip condition then using equation (i), we have

$$Q = \pi \int_0^R r^2 \left(-\frac{dw}{dr}\right) dr = \pi \int_0^R r^2 f(\tau) dr \quad \text{...(ix)}$$

Now the shear stress at the wall τ_R, considering p as pressure and equation (viii) is:

$$\tau = -\frac{r}{2}\frac{dp}{dz}, \text{ and } \tau_R = -\frac{R}{2}\frac{dp}{dz} \quad \text{...(x)}$$

From equation (ix) and (x) we have,

$$Q = \pi \int_0^{\tau_R} \frac{\tau^2 R^2}{\tau_R} - f(\tau) d\tau = \frac{\pi R^3}{\tau^3{}_R} \int_0^{\tau_R} \tau^2 f(\tau) d\tau \quad \text{...(xi)}$$

Now put the value of $f(\tau)$ from equation (i) in to the equation (xi), we gets

$$Q = \frac{\pi R^3}{\tau_R{}^3} \int_0^{\tau_R} \tau^2 \left(\frac{\tau - \tau_0}{\mu}\right) d\tau$$

or

$$Q = \frac{\pi R^3}{\mu \tau_R^3} \int_0^{\tau_R} (\tau^3 - \tau^2 \tau_0) d\tau = \frac{\pi R^3 \tau_R}{\mu}\left[\frac{1}{4} - \frac{\tau_0}{\tau_R}\right] \quad \text{...(xii)}$$

Now then the pressure gradient using equation (xii) may be written as:

$$\frac{dp}{dz} = -\frac{2}{R}\left(\tau_0 + \frac{4\mu Q}{\pi R^3}\right)$$

$$\frac{dp}{dz} = -\frac{2}{R}\left(\tau_0 + \frac{4\mu Q}{\pi R^3}\right)$$

Integrating with respect to z with the condition $p = p_i$ at $z = 0$ and $p = p_0$ at $z = L$ then

$$p_i - p_0 = -\frac{2\tau_0}{R_0} \int_0^L \left(\frac{R}{R_0}\right)^{-1} dz - \frac{4\mu Q}{\pi R^4} \int_0^L \left(\frac{R}{R_0}\right)^{-4} dz$$

or

$$\frac{p_i - p_0}{Q} = -\frac{2\tau_0}{R_0 Q} \int_0^L \left(\frac{R}{R_0}\right)^{-1} dz - \frac{4\mu}{\pi R^4} \int_0^L \left(\frac{R}{R_0}\right)^{-4} dz \quad \text{...(xiii)}$$

Let resistance to flow is λ are may be expressed as:

$$\lambda = \frac{p_i - p_0}{Q}$$

and

$$f = -\frac{2\tau_0}{R_0 Q}, \quad h = -\frac{4\mu}{\pi R^4}, \quad \text{...(xiv)}$$

then equation (xiii) become:

$$\lambda = -f\int_0^L \left(\frac{R}{R_0}\right)^{-1} dz - h\int_0^L \left(\frac{R}{R_0}\right)^{-4} dz \qquad \text{…(xv)}$$

It is important to define that where each of these abnormal segments start and end. The start of each portion is given by x_i and end is given by y_i.

Taking this into account and using the position of the start and end of the typical section allow the use of equation (xiv) then we have,

$$\lambda = -f\left[\int_0^{\alpha_i} dz + \sum_{i=1}^{m}\int_{\alpha_i}^{\beta_i}\left(\frac{R}{R_0}\right)^{-1} dz + \sum_{i=1}^{m-i}\int_{\alpha_i}^{\beta_i} dz + \int_{\beta_0}^{L} dz\right]$$

$$-h\left[\int_0^{\alpha_i} dz + \sum_{i=1}^{m}\int_{\alpha_i}^{\beta_i}\left(\frac{R}{R_0}\right)^{-4} dz + \sum_{i=1}^{m-i}\int_{\alpha_i}^{\beta_i} dz + \int_{\beta_0}^{L} dz\right] \qquad \text{…(xvi)}$$

Let

$$I_1 = \left[\sum_{i=1}^{m}\int_{\alpha_i}^{\beta_i}\left(\frac{R}{R_0}\right)^{-1} dz\right] \qquad \text{…(xviia)}$$

$$I_2 = \left[\sum_{i=1}^{m}\int_{\alpha_i}^{\beta_i}\left(\frac{R}{R_0}\right)^{-4} dz\right] \qquad \text{…(xviib)}$$

put the value of I_1 and I_2 in equation (xvi) the we have

$$\lambda = -f\left[\sum_{i=1}^{m+1} d_i + I_1\right] - h\left[\sum_{i=1}^{m+1} d_i + I_2\right]$$

or

$$\lambda = -(f+h)\left(\sum_{i=1}^{m+1} d_i\right)(f\,I_1 + h\,I_2) \qquad \text{…(xviii)}$$

If there is no abnormal segment then

$$\lambda_N = -(f+h)L$$

hence,

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = \frac{\sum_{i=1}^{m+1} d_i}{L} + \frac{fI_1 + hI_2}{L} \qquad \text{…(xix)}$$

Now from equation (ii) for $\frac{R}{R_0}$ it is possible to solve analytically for the integrals given by equation (xviia) and (xviib).

The solution process we will have a simple form if we are defining the following variable:

$$a = 1 - \frac{\delta}{2R_0},\ b = \frac{\delta}{2R_0},\ \phi = \pi - \frac{2\pi}{l_i}\left(z - \alpha_i - \frac{l_i}{z}\right),\ c = \frac{a}{b}.$$

As $z = x_i \Rightarrow \phi = 2\pi$ and $z = y_i \Rightarrow \phi = 0$

Putting all these value in equation (ii), then we have

$$I_1 = \frac{l_0}{2\pi}\int_0^{2\pi}\frac{d\phi}{a + b\cos\phi} = \frac{l_0}{2\pi}\frac{1}{\sqrt{a^2 + b^2}} \qquad \text{…(xx)}$$

and

$$I_2 = \frac{l_0}{2\pi}\int_0^{2\pi}\frac{d\phi}{(a+b\cos\phi)^4},\ I_1 = \frac{l_0}{2\pi}\frac{1}{(a^2+b^2)}\left(1-12v+10v^2-20v^3\right) \qquad \ldots\text{(xxi)}$$

where $$v = \frac{-c+\sqrt{c^2-1}}{2\sqrt{c^2-1}}$$

Thus the value of $\bar{\lambda}$ is obtained by equation (xix)

IV. RESULT AND DISCUSSION

The influences of yields stress, viscosity and flux of blood on the resistance to flow $\bar{\lambda}$ for flow through vessels containing segment that vary from δ_1/R_0 and δ_2/R_0 are consider through fig 2.

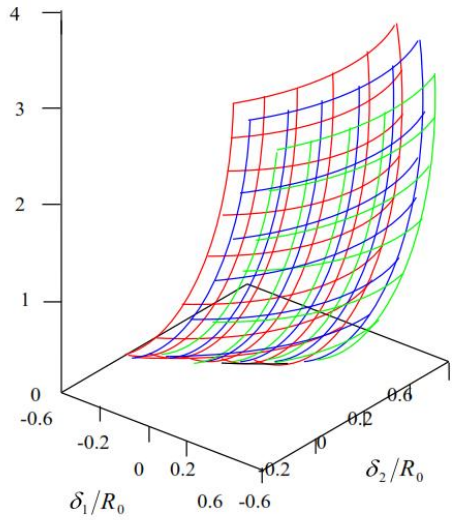

Fig. 2 Variation in resistance flow ratio with yield stress ($red\ curve = I$, $blue\ curve = II$ and $green\ curve = III$)

Figure 2, show the variation in resistance to flow ratio $(\bar{\lambda})$with yield stress (τ_0). I curve shows the variation in $\bar{\lambda}$ at yield stress 0. II curve shows the variation in $\bar{\lambda}$ at yield stress 0.005. III curve shows the variation in $\bar{\lambda}$ at yield stress 0.01. We see that if the value of yield stress increased then the value of resistance to flow ratios decreased and go close to unity.

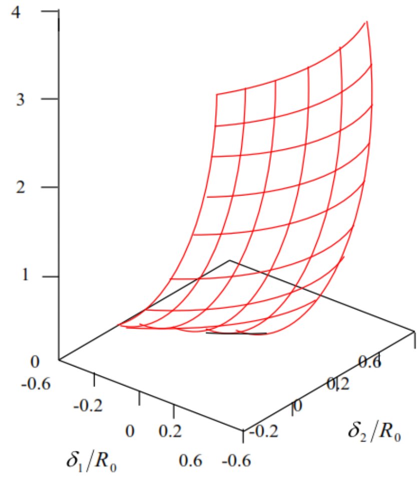

Fig. 3 Variation in resistance flow ratio with viscosit

Figure 3 shows the variation in resistance to flow ratio with the viscosity (μ). The graph shows there are no significant variation in $\bar{\lambda}$ for $\mu = 3.45\times10^{-3}, 4\times10^{-3}$ and. $4.55\times10^{-3}\,Pa.s$. Figure 4, shows the variation in resistance to flow ratio $(\bar{\lambda})$ with flux Q.

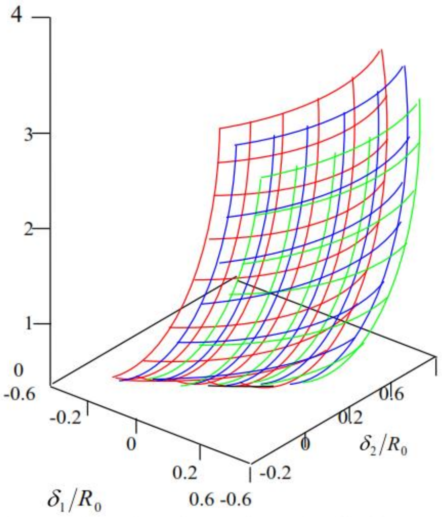

Fig. 4 Variation in resistance flow ratio with flux

I curve shows the variation in $\bar{\lambda}$ at $Q=1$. II curve shows the variation in $\bar{\lambda}$ at $Q=10$. III curve shows the variation in $\bar{\lambda}$ at $Q=100$. We see that if the value of flux (Q) increased then the value of resistance to flow ratios $(\bar{\lambda})$ increased.

V. CONCLUSION

The radius of the artery is 0.45mm, the length of the artery is 45mm and the length of artery at abnormalities is 15mm taken in this work. The valve of $\bar{\lambda}$ is vary from 0.525 to 3.06 and the magnitude of resistance to flow ratio is greatest for yield stress and least for flux. Increasing the yield stress τ_0 and decreasing the viscosity μ or flux Q the resistance flow ratio become close to unity.

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