FLEXURAL TORSIONAL BEHAVIOUR OF THIN WALLED MONO SYMMETRIC BOX GIRDER STRUCTURES

Chidolue C.A.[1] and Osadebe N.N.[2]
1 Dept of Civil Engineering, Nnamdi Azikiwe University, Awka, Nigeria
chidoluealfred@gmail.com
2 Dept of Civil Engineering, University of Nigeria, Nsukka, Nigeria
nkemuamaka@yahoo.com

ABSTRACT
Thin-walled mono symmetric box girder structures are commonly found in the form of trapezoidal cross sections of either concrete or steel. Such structures resist eccentric vertical loads in bending action and torsion. The torsional components of eccentric loads on such structures give rise to pure torsion (Saint Venant torsion), distortion and flexure about the non symmetric axis of the box girder section. In order to provide an improved understanding of the complex interactions between these strain fields, this paper examined the interaction between the torsional strain mode and flexural strain mode and derived a general differential equations of equilibrium for flexural-torsional analysis of mono symmetric box girder structures. In addition, the derived equations were used to analyze a double cell simply supported mono symmetric box girder section to obtain flexural and torsional deformations. Investigations show that there is no interaction between flexural and torsional strain modes in the axis of symmetry of the box girder structure, hence torsion is independent of flexural deformation. However, in the non symmetric axis there is strong interaction between flexure and torsion resulting to the coupling of the equilibrium equations. The results show that the maximum (mid span) flexural deformation was about twenty six times more than torsional deformation.

***KEYWORDS*:** Flexure, torsion, deformation, thin walled, mono symmetric box girder, Vlasov's theory.

I. INTRODUCTION

A thin-walled structure is one which is made from thin plates joined along their edges. The plate thickness however is small compared to other cross sectional dimensions which are in turn often small compared with the overall length of the member or structure. Thin walled structures have gained special importance and notably increased application in recent years. The wide use of these thin walled structures is due to their great carrying capability and reliability and to the economic advantage they have over solid (column and beam) structures. Initially design of box girder bridges is related to the design of plate girder bridges. However, such design knowledge does not contain important primary conditions of cross sectional deformations such as warping and distortion. The application of cross sectional deformation equations formulated by Vlasov [1] and Dabrowski [2], has opened a new way to analyze the torsional and distortional effects of loads on such girders.

The objective of this study is to derive a set of differential equations governing the flexural-torsional behaviour of thin- walled box girder bridge structures of mono symmetric cross section on the basis of Vlasov's theory and to apply the obtained differential equations in the analysis of single cell mono symmetric box girder section to obtain flexural and torsional deformations.

Research [3] has shown that a mono symmetric thin walled box girder has three strain modes interactions: torsion interacts with distortion and each of these strain modes interacts with flexure about the non axis of symmetry. Thus we have torsional-distortional interaction, flexural-torsional interaction, and flexural-distortional interaction. In this work, the interaction of flexural strain mode

about the non axis of symmetry with torsional strain mode of a mono symmetric box girder structure is examined.
In section two of this report, a literature review of past work on thin-walled box girder structure is presented, followed by a discussion on Vlasov's stress strain relationships (in section three) which form the basis for energy formulation of the equilibrium equations in section four. Generation of strain modes diagram, which is vital part of this work, and the evaluation of Vlasov's coefficients of differential equations of equilibrium are presented in sections five and six. In section seven the specific differential equations of flexural-torsional equilibrium for mono symmetric box girder example frame are derived and used in section eight to obtain the flexural and torsional deformations. Discussion of results and conclusions are presented in sections nine and ten respectively.

II. REVIEW OF PAST WORK

The curvilinear nature of box girder bridges along with their complex deformation patterns and stress fields have led designers to adopt approximate and conservative methods for their analysis and design. Recent literatures: Hsu et al [4], Fan and Helwig [5], Sennah and Kennedy [6], on straight and curved box girder bridges deal with analytical formulations to better understand the behaviour of these complex structural systems. Few authors, Okil and El-tawil [7], Sennah and Kennedy [6], have undertaken experimental studies to investigate the accuracy of existing methods. Before the advent of Vlasov's 'theory of thin-walled beams' [1], the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure
Several investigators; Bazant and El-Nimeiri [8], Zhang and Lyons [9], Boswell and Zhang [10], Usuki [11], Waldron [12], Paavola [13], Razaqpur and Lui [14], Fu and Hsu [15], Tesar [16], have combined thin-walled beam theory of Vlasov and the finite element technique to develop a thin walled box element for elastic analysis of straight and curved cellular bridges. Osadebe and Chidolue [17], [18], [19], Eze [20] obtained fourth order differential equations of torsional-distortional equilibrium for the analysis of mono symmetric box girder structures using Vlasov's theory with modifications by Varbanov [21].
Various theories were therefore postulated by different authors examining methods of analysis, both classical and numerical. A few others however carried out tests on prototype models to verify the authenticity of the theories. At the end of it all, we concluded that Vlasov's theory captures all peculiarities of cross sectional deformations such as warping, torsion, distortion etc, and is therefore adopted in this work.

III. VLASOV'S STRESS – STRAIN RELATIONS

The longitudinal warping and transverse (distortional) displacements given by Vlasov [1] are

$$u(x,s) = U(x)\varphi(s) \quad \text{and} \quad v(x,s) = V(x)\psi(s) \tag{1}$$

The displacements may be represented in series form as;

$$u(x,s) = \sum_{i=1}^{m} U_i(x)\varphi_i(s) \quad \text{and} \quad v(x,s) = \sum_{k=1}^{n} V_k(x)\psi_k(s) \tag{2}$$

where, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the box girder frame. $\varphi_i(s)$ and $\psi_k(s)$ are elementary displacements of the strip frame, respectively out of the plane (m displacements) and in the plane (n displacements). These displacements are chosen among all displacements possible, and are called the generalized strain coordinates of a strip frame.
From the theory of elasticity the strains in the longitudinal and transverse directions are given by;

$$\frac{\partial u(x,s)}{\partial x} = \sum_{i=1}^{m} U_i'(x)\varphi_i(s) \quad \text{and} \quad \frac{\partial v(x,s)}{\partial x} = \sum_{k=1}^{n} V_k'(x)\psi_k(s) \tag{3}$$

The expression for shear strain is $\gamma(x,s) = \dfrac{\partial u}{\partial s} + \dfrac{\partial v}{\partial x}$

or $$\gamma(x,s)=\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x) \tag{4}$$

Using the above displacement fields, φ_i *and* ψ_i, and basic stress-strain relationships of the theory of elasticity the expressions for normal and shear stresses become:

$$\sigma(x,s)=E\frac{\partial u(x,s)}{\partial x}=E\sum_{i=1}^{m}\varphi_i(s)U_i'(x) \tag{5}$$

and $$\tau(x,s)=G\gamma(x,s)=G\left[\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x)\right] \tag{6}$$

Transverse bending moment generated in the box structure due to distortion is given by;

$$M(x,s)=\sum_{k=1}^{n}M_k(s)V_k(x) \tag{7}$$

where $M_k(s)$ = bending moment generated in the cross sectional frame of unit with due to a unit distortion, V(x) = 1

IV. ENERGY FORMULATION OF THE EQUILIBRIUM EQUATIONS

The potential energy of a box structure under the action of a distortional load of intensity q is given by:

$$\Pi = U + W_E \tag{8}$$

Where,
Π = the total potential energy of the box structure,
U = Strain energy,
W_E = External potential or work done by the external loads.
From strength of materials, the strain energy of a structure is given by

$$U=\frac{1}{2}\int_L\int_S\left[\left(\frac{\sigma^2(x,s)}{E}+\frac{\tau^2(x,s)}{G}\right)t(s)+\frac{M^2(x,s)}{EI_{(s)}}\right]dxds \tag{9}$$

Work done by external load is given by;

$$W_E = qv(x,s)dxds=\int_s\int_x q\sum V_h(x)\varphi_h(s)dsdx=\int_x\sum q_hV_hdx \tag{10}$$

Substituting eqns (9) and (10) into eqn.(8) we obtain that,

$$\Pi=\frac{1}{2}\left[\int_L\int_S\left[\frac{\sigma^2(x,s)}{E}+\frac{\tau^2(x,s)}{G}\right]t(s)+\frac{M^2(x,s)}{EI(s)}-qv(x,s)\right]dxds \tag{11}$$

where,
$\sigma(x,s)$ = Normal stress
$\tau(x,s)$ = Shear stress
$M(x,s)$ = Transverse distortional bending moment
q = Line load per unit area applied in the plane of the flange plate
$I_{(s)}=\frac{t^3(s)}{12(1-\nu^2)}$ = Moment of inertia of the plates
E = Modulus of elasticity
G = Shear modulus

ν = poisson ratio
t = thickness of plates (assumed uniform)
Substituting eqns (1), (5), (6), and (7) into eqn. (11) and simplifying, noting that $t(s)ds = dA$ we obtain the potential energy of the box structure as follows.

$$\Pi = \frac{E}{2}\sum a_{ij}U_i'(x)U_j'(x)dx + \frac{G}{2}\left[\sum b_{ij}U_i(x)U_j(x) + \sum c_{kj}U_k(x)V_j'(x)\right]dx$$
$$+\frac{G}{2}\left[\sum c_{ih}U_i(x)V_h'(x) + \sum r_{kh}V_k'(x)V_h'(x)\right]dx + \frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx - \sum q_h V_h dx \qquad (12)$$

where the (Vlasov's) coefficients are given as follows:

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA \qquad (a)$$
$$b_{ij} = b_{ji} = \int \varphi_i'(s)\varphi_j'(s)dA \qquad (b)$$
$$c_{kj} = c_{jk} = \int \varphi_k'(s)\psi_j(s)dA \qquad (c)$$
$$c_{ih} = c_{hi} = \int \varphi_i'(s)\psi_k(s)dA \qquad (d) \qquad (13)$$
$$r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; \qquad (e)$$
$$s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad (f)$$
$$q_h = \int q\psi_h ds \qquad (g)$$

The governing equations of flexural-torsional equilibrium are obtained by minimizing the energy functional eqn. (12), with respect to its functional variables u(x) and v(x) using Euler Lagrange technique, Elgolts [20]. Minimizing with respect to u(x) we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k'(x) = 0 \qquad (14)$$

Minimizing with respect to v(x) we have;

$$\sum c_{ih}U_i'(x) - \kappa\sum s_{hk}V_k(x) + \sum r_{kh}V_k''(x) + \frac{1}{G}\sum q_h = 0 \qquad (15)$$

where $\kappa = \frac{E}{G} = 2(1+\nu)$

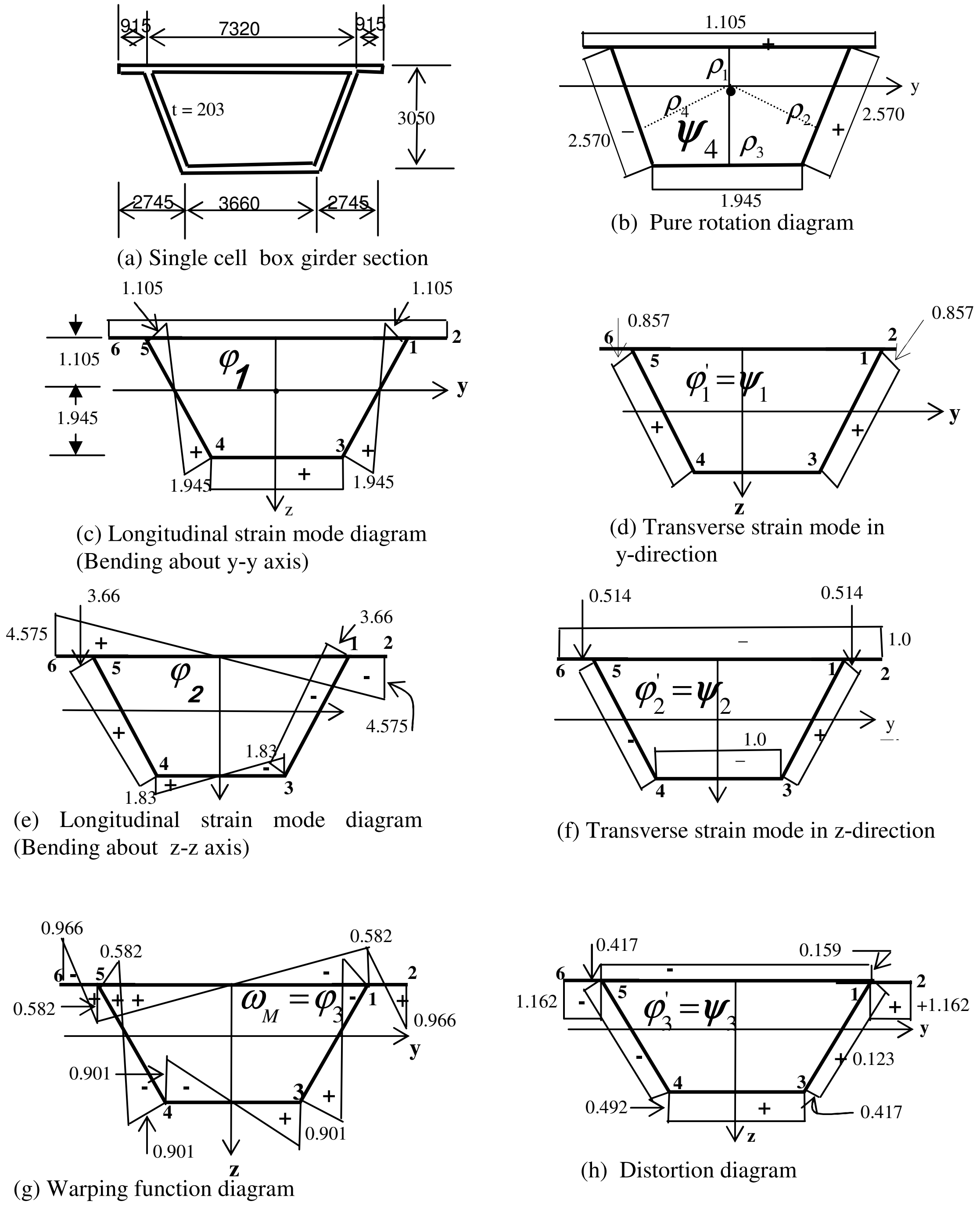
Equations (14) and (15) are Vlasov's generalized differential equations of distortional equilibrium for a box girder. They are presented in matrix form as follows:

$$\kappa\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{Bmatrix} U_1'' \\ U_2'' \\ U_3'' \end{Bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} - \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix}\begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{Bmatrix} = 0 \qquad (16a)$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix}\begin{Bmatrix} U_1' \\ U_2' \\ U_3' \end{Bmatrix} - \kappa\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{41} & s_{43} & s_{44} \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} + \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}\begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4'' \end{Bmatrix} + \frac{1}{G}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0 \qquad (16b)$$

V. Strain Mode Diagrams and Evaluation of Vlasov's Coefficients

Fig.1 shows a single cell mono symmetric box girder structure and its strain mode diagrams used for numerical example. The coefficients $a_{ij}, b_{ij}, c_{kj}, c_{ih}$ and r_{kh}, of the governing equations of equilibrium are computed with the aid of Morh's integral chart using the strain modes diagrams. The procedure for obtaining strain modes diagrams is available in literatures [17], [23]. The summary of the coefficients for the single cell mono symmetric box girder example problem is given on Table 1.



(a) Single cell box girder section
(b) Pure rotation diagram
(c) Longitudinal strain mode diagram (Bending about y-y axis)
(d) Transverse strain mode in y-direction
(e) Longitudinal strain mode diagram (Bending about z-z axis)
(f) Transverse strain mode in z-direction
(g) Warping function diagram
(h) Distortion diagram

Fig.1 Generalized strain modes for single cell mono-symmetric box girder frame

Table 1: Summary of Vlasov's coefficients for the single cell mono symmetric box girder frame

$a_{ij} = a_{ji}$	$b_{ij} = b_{ji}$	$c_{kj} = c_{jk}$	$c_{ih} = c_{hi}$	$r_{kh} = r_{hk}$
$a_{11} = 6.453$ $a_{12} = 0$ $a_{13} = 0$ $a_{22} = 25.050$ $a_{23} = -0.270$ $a_{33} = 0.757$ $s_{33} = 0.261 * I_s$	$b_{11} = 1.060$ $b_{12} = 0$ $b_{13} = 0$ $b_{22} = 2.982$ $b_{23} = -1.066$ $b_{33} = 1.407$	$c_{11} = 1.060$ $c_{12} = 0$ $c_{13} = 0$ $c_{22} = 2.982$ $c_{23} = -1.066$ $c_{33} = 1.407$ $c_{41} = 0$ $c_{42} = -1.561$ $c_{43} = 1.265$	$c_{11} = 1.060$ $c_{12} = 0$ $c_{13} = 0$ $c_{14} = 0$ $c_{22} = 2.982$ $c_{23} = -1.066$ $c_{33} = 1.407$ $c_{24} = -1.561$ $c_{34} = 1.265$	$r_{11} = 1.060$ $r_{12} = 0$ $r_{13} = 0$ $r_{14} = 0$ $r_{22} = 2.982$ $r_{23} = -1.066$ $r_{24} = -1.561$ $r_{33} = 1.407$ $r_{34} = 1.265$ $r_{44} = 14.616$

VI. EVALUATION OF DISTORTIONAL BENDING MOMENT COEFFICIENTS S_{hk}

The distortional bending moment coefficients s_{hk}, given by eqn. (13f) depend on the bending deformation of the strip frame characterized by the bending moment, M_k (for k = 1, 2, 3, 4). To compute the coefficients we need to construct the diagram of the bending moments due to strain modes, $\psi_1, \psi_2, \psi_3, and\ \psi_4$. Incidentally, ψ_1, ψ_2 and ψ_4 strain modes do not generate distortional bending moment on the box girder structure as they involve pure bending and pure rotation. Only ψ_3 strain mode generates distortional bending moment which can be evaluated using the distortion diagram for the relevant cross section. Consequently the relevant expression for the coefficient becomes:

$$s_{hk} = s_{kh} = \frac{1}{E} \int_s \frac{M_3(s) M_3(s)}{EI_s} \qquad (19)$$

where $M_3(s)$ is the distortional bending moment of the relevant cross section due to strain mode 3. The procedure for evaluation of distortional bending moments is given in literatures [17], [23]. Fig. 2 shows the distortional bending moment for evaluation of S_{hk} for the single cell mono symmetric frame of Fig. 1(a). The computed value of $s_{hk} = s_{33}$ for the single cell mono symmetric frame example was $s_{33} = 0.261 * I_S$.

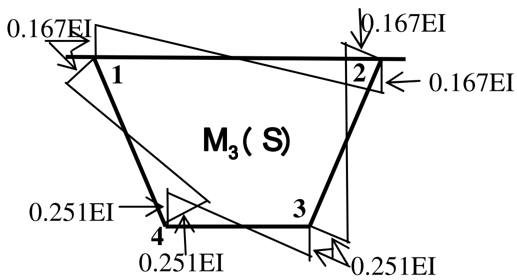


Fig. 2 : Bending moment due to distortion of single cell mono-symmetric section

VII. FLEXURAL-TORSIONAL DIFFERENTIAL EQUATIONS OF EQUILIBRIUM

The relevant coefficients for flexural – torsional equilibrium equations are those involving strain modes 2 and 4, i.e., a_{22}, b_{22}, c_{22}, $c_{24} = c_{42}$; r_{22}, $r_{24} = r_{42}$, and r_{44}. All other coefficients are zero. Substituting these coefficients into the matrix eqns (16a) and (16b) and multiplying out we obtain

$$ka_{22}U_2'' - b_{22}U_2 - c_{22}V_2' - c_{24}V_4' = 0 \quad (17)$$

$$c_{22}U_2' + r_{22}V_2'' + r_{24}V_4'' = -\frac{q_2}{G} \quad (18)$$

$$c_{42}U_2' + r_{42}V_2'' + r_{44}V_4'' = -\frac{q_4}{G} \quad (19)$$

Simplifying further we obtain the coupled differential equations of flexural-torsional equilibrium for mono symmetric sections as follows.

$$\begin{aligned} &V_4'' = K_1 \quad (a) \\ &\alpha_1 V_2^{iv} + \alpha_2 V_4^{iv} - \beta_1 V_4'' = K_2 \quad (b) \end{aligned} \quad (20)$$

where,

$$\alpha_1 = \kappa a_{22}c_{42}; \quad \alpha_2 = ka_{22}r_{44}; \quad \beta_1 = \left(b_{22}r_{44} - c_{24}c_{42}\right);$$

$$K_1 = \left(\frac{c_{22}}{c_{42}r_{24} - c_{22}r_{44}}\right)\frac{\overline{q}_4}{G} - \left(\frac{c_{42}}{c_{42}r_{24} - c_{22}r_{44}}\right)\frac{\overline{q}_2}{G}; \quad K_2 = b_{22}\frac{\overline{q}_4}{G}$$

$$\overline{q}_2 = q_2 * b; \quad \overline{q}_4 = q_4 * b$$

b = width of the top flange

VIII. FLEXURAL-TORSIONAL ANALYSIS OF SINGLE CELL MONO SYMMETRIC FRAME

In this section the solutions of the differential equations of equilibrium eqns.(20) are obtained for the single cell mono symmetric box girder section shown in Fig.1a. Live loads are considered according to AASHTO-LRFD [24], following the HL-93 loading: uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A 50m span simply supported bridge deck structure was considered. The obtained torsional loads were;

$\overline{q}_2 = 0.00KN, \quad \overline{q}_4 = 1446.505KN$.

The governing equations of equilibrium are eqns. (20)
The values of the relevant coefficients from Table 1 are

$$a_{22} = 25.05; \quad b_{22} = c_{22} = r_{22} = 2.982$$

$$c_{24} = c_{42} = r_{24} = r_{42} = -2.515$$

$$r_{44} = 14.616; \quad s_{33} = 0.261 * 6.667 * 10^{-4} = 1.740 * 10^{-4}$$

The coefficients of the governing equations are

$$\alpha_1 = -157.502; \quad \alpha_2 = 915.327; \quad \beta_1 = 49.910$$

$$K_1 = 4.4932 * 10^{-4}; \quad K_2 = -1.206 * 10^{-5}$$

$$E = 24 * 10^9 N/m^2; \quad G = 9.6 * 10^9 N/m^2, \quad k = 2.5$$

Substituting the coefficients into eqns (20) we obtain

$$\begin{aligned} &V_4'' = -1.206x10^{-5} \quad (a) \\ &-157.502V_2^{iv} + 915.327V_4^{iv} - 49.910V_4'' = 4.4932 * 10^{-4} \quad (b) \end{aligned} \quad (21)$$

Integrating by method of trigonometric series with accelerated convergence we have,

$$V_2(x) = 79.73*10^{-3} Sin\frac{\pi x}{50}; \quad V_4(x) = 3.055*10^{-3} Sin\frac{\pi x}{50} \tag{22}$$

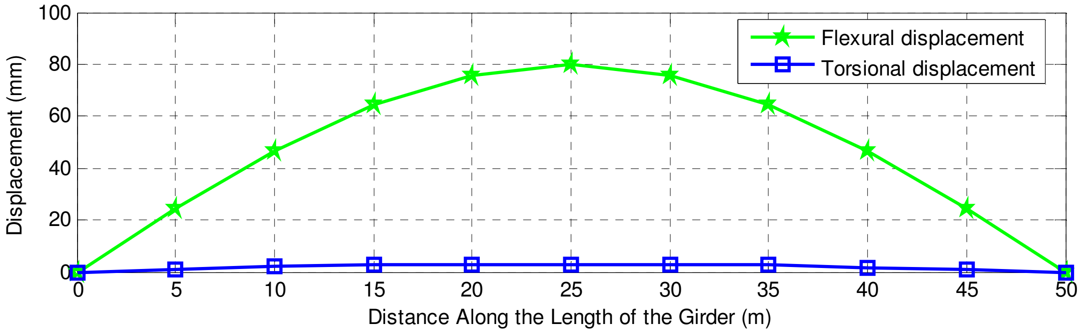


Fig. 3 Variation of flexural and torsional displacements along the length of the girder

IX. Discussion of Results

From the computation of Vlasov's coefficients for the mono symmetric box girder structure which form a major part of this work, it was ascertained that a mono symmetric box girder frame has three strain modes interactions: torsional-distortional interaction, flexural-distortional interaction and flexural-torsional interaction. Where there are no interactions between two or more strain modes the values of the relevant coefficients was found to be zero. For example, the value of the coefficient a_{13} was zero indicating that there was no interaction between bending strain mode 1 represented by φ_1 diagram and distortional strain mode 3 represented by φ_3 diagram In such a situation, the analysis of the box girder frame for any one of the strain modes can be carried out independent of the other strain modes.

The derived governing differential equations of flexural-torsional equilibrium for mono symmetric box girder structures are given by eqn. (20). They are applicable to both single cell and multi cell profiles. Eqn. (20a) shows that torsional deformations are independent of flexural deformations on the axis of symmetry where flexure does not interact with torsion. However, on the non symmetric axis of the box girder where torsion interacts with flexure, eqn. (20b) shows that torsional deformations can not be obtained independent of flexural deformation.

For the single cell mono symmetric box girder section shown in Fig.1a, the expressions for flexural and torsional displacements obtained by integration of eqn. (20) are given by eqn. (22), and graphically presented in Fig. 3. Thus, Fig.3 shows the flexural and torsional deformations of the box girder frame along the girder length. The maximum (mid span) flexural deformation was 80mm while the mid span torsional deformation was 3mm. Thus, the flexural deformation was more than twenty six times that of torsional deformation (in a simply supported box girder span of 50m).

X. Conclusions

The obtained governing differential equations of flexural-torsional equilibrium consists of a second order equation involving only torsional displacement as unknown and a fourth order coupled linear differential equation having flexural and torsional deformations as unknowns. There is no interaction between flexure and torsion in the axis of symmetry, hence torsion is independent of flexural deformation as indicated by eqn. (20a). However, in the non symmetric axis there is strong interaction between flexure and torsion resulting to coupled eqn.(20b).

The results show that the maximum (mid span) flexural deformation was about twenty seven times that of torsional deformation, for a simply supported box girder span of 50m.

References

[1] V.Z.Vlasov, *Thin-walled space structures* (Gosstrojizdat, Moscow, 1958).

[2] R. Dabrowski, "Curved thin-walled girders", Translation No. 144, Cement and Concrete Association, London, (1968).

[3] C.A. Chidolue, Torsional-distortional analysis of thin-walled box girder bridges using Vlasov's theory, Ph.D. thesis 2012, University of Nigeria, Nsukka.
[4] Y.T. Hsu, C.C. Fu and D.R. Shelling, EBEF method for distortional analysis of steel box girder bridges, *Journal of Structural Engineering, 121(3) 1995, 557-566*
[5] Z. Fan, and T.A. Helwig, Distortional loads and brace forces in steel box girders, *Journal of. Structural Engineering, 128(6),* 2002, 710-718.
[6] K.M. Sennah, and J.B. Kennedy, Literature review in analysis of box girder bridges, *Journal of Bridge Engineering, 7 (2),* 2002, 134-140.
[7] A.M. Okeil, and S. El-Tawil, Warping stresses in curved box girder bridges: case study, *Journal of Bridge Engineering, 9(5)* , 2004, 487- 496.
[8] Z.P. Bazant and M. El-Nimeiru, Stiffness method for curved box girders at initial stress, *Journal of Structural Division, 100(10),* 1974, 2071-2090.
[9] S.H. Zhang and L.P.R. Lyons, Thin-walled box beam finite element for curved bridge analysis, *Comput. Struct. Journal*, 18(6), 1984, 1035-1046.
[10] L.F. Boswell and S.H. Zhang, The effect of distortion in thin-walled box-spine beams, *International Journal of Solid Structures, 20(9/10),* 1984, 845-852.
[11] T. Usuki, The Theory of Curved Multi-Cell Box Girder Bridges Under Consideration of Cross-Sectional Distortion, *Struct. Eng./Earthquake Eng. Journal, 4(2),* 1987, 277-287
[12] P. Waldron, The Significance of warping torsion in the design of straight concrete box-girder Bridges, *Canadian Journal of Civil Engineering, 15(5)* 1988, 879-889
[13] J. Paavola, *Study of curved Thin-Walled Girders,* doctoral dissertation., Helsinki University of Technology, Espoo, Finland, PhD, 1990.
[14] A.G. Razaqpur, and H.G. Li, Analysis of multi-branch multi-cell box girder bridges, *Proc.3rd Int. Conf. on Short and Medium Span Bridges*, Toronto, 1990, 153-164
[15] C.C. Fu, and Y.T. Hsu, The Development of an Improved Curvilinear Thin-Walled Vlasov Element, *Computational Structures Journal, 54(1),* 1995, 147-159.
[16] A. Tesar, Shear Lag in the Behaviour of Thin-Walled Box Bridges, *Comput. Struct. Journal, 59(4),* 1996, 607-612.
[17] N.N. Osadebe and C.A. Chidolue, Torsional-distortional response of thin-walled mono symmetric box girder structures, *International journal of engineering research and applications*, 2(3), 2012, 814-821.
[18] N.N. Osadebe and C.A. Chidolue, Response of double cell mono symmetric box girder structure to torsional-distortional deformations, *International journal of engineering and advanced technology*, 1(4), April 2012, 285-292
[19] N.N. Osadebe and C.A. Chidolue, Effect of flexural-torsional-distortional interactions on the behaviour of thin-walled mono symmetric box girder structures, *International journal of engineering research and applications*, 2(3), 2012, 1685-1693.
[20] Ezeh J.C., Buckling behaviour of axially compressed multi cell doubly symmetric thin-walled column using Vlasov's theory, *International journal of engineering*, 4(2), 2010,179-194
[21] C.P. Varbanov, *Theory of elasticity* (Technika Press Sofia, 4th Edition, 1976).
[22] L. Elsgolt, *Differential Equations and the Calculus of Variation*, (MIR publishers, Mosco,1980, translated from the Russian by George Yankovsky.)
[23] V.G. Rekech, *Static theory of thin-walled space structures*, MIR publishers, Mosco, (1978)
[24] American Association of State Highway and Transportation Officials (AASHTO), *Load and Resistance Factor Design*, LRFD, Bridge Design Specifications Washington , D.C, 1998.

Author Biography

Chinenye Alfred Chidolue was born in 1952 in Nigeria. He obtained B.Eng (Civil) degree from the University of Zambia , M.Eng degree (Civil and Structural) from University of Sheffield and Ph.D.degree in Structural Engineering is being awaited from University of Nigeria Nsukka. He is currently a Senior lecturer in civil engineering department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.